Tensor Product Model Transformation-based Sliding Surface Design

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Abstract: Motion control has been a fruitful ground for applying Variable Structure Systems (VSS) theory. This paper provides an assessment of the state of the art of the relevant theoretical results for sliding mode control. The design of a sliding-mode controller consists of three main steps. First step is the design of the sliding surface, the second step is the design the control law which holds the system trajectory on the sliding surface, and the third and key step is the chattering-free implementation. The main contribution of that paper is a new method for sliding surface sector design based on tensor product (TP) model transformation to reduce the chattering.

Keywords: Sliding mode control, sliding sector design, Tensor product, friction compensation

1 Introduction

Sliding mode has been introduced in the late 1970's [1, 2] for highly coupled nonlinear dynamics, with unknown system parameters and disturbances. In the early 1980's, sliding mode was further introduced for the control of induction motor drives [3]. Its utility in this hybrid discipline, consisting of power electronics and motion control, is to provide direct switching strategy [4] to the power electronics devices such that, in spite of the nonlinear dynamics of the induction motor, the control design is decomposed into a nonlinear control synthesis problem, and a linear control design problem of reduced order. These early applications of sliding mode indicated the versatility of the underlying control principles in the design of feedback control systems for motion control, regardless of the origin or the nature of the particular system performance specifications and design goals.

This initial works were followed by a large number of research papers in robotic manipulator control and in motor drive control. References can be found in [5]. In some of these works, experimental results were published [6, 7, 8, 9]. However, despite of the theoretical predictions of superb closed loop system performance of sliding mode, some of the experimental works indicated that sliding mode in practice has limitations due to the need of high sampling frequency to reduce the high frequency oscillation phenomenon about the sliding mode manifold -- collectively referred to as ‘chattering’. Among these experimental works, a few succeeded to show closed loop system behaviour, which were predicted by theory. Those who failed to manage the experimental designs successfully concluded that chattering is a major problem in realizing sliding mode control in practice. The usual sources of chattering are the limited switching frequency and the un-modeled dynamics, which are ignored in the theoretical design steps [10]. A detailed simulation of the whole system including controller and discrete semiconductor switches can be an important middle step in the chattering free implementation of sliding mode. Another promising method for reducing chattering is the sliding sector design [20], which is in the focus of that paper. A tensor product model transformation is proposed for design a sliding surface.

The tensor product (TP) model form is a dynamic model representation whereupon Linear Matrix Inequality (LMI) based control design techniques [11]–[13] can immediately be executed. It describes a class of Linear Parameter Varying (LPV) models by the convex combination of linear time invariant (LTI) models, where the convex combination is defined by the weighting functions of each parameter separately. An important advantage of the TP model forms is that the convex hull of the given dynamic LPV model can be determined and analysed by one variable weighting functions. Furthermore, the feasibility of the LMIs can be considerably relaxed in this representation via modifying the convex hull of the LPV model.

The TP model transformation is a recently proposed numerical method to transform LPV models into TP model form [14], [15]. It is capable of transforming different LPV model representations (such as physical model given by analytic equations, fuzzy, neural network, genetic algorithm based models) into TP model form in a uniform way. In this sense it replaces the analytical derivations and affine decompositions (that could be a very complex or even an unsolvable task), and automatically results in the TP model form. Execution of the TP model transformation takes a few minutes by a regular Personal Computer. The TP model transformation minimizes the number of the LTI components of the resulting TP model. Furthermore, the TP model transformation is capable of resulting different convex hulls of the given LPV model.

The rest of the paper is organized as follows: Section 2 describes the main steps of sliding mode control design including basis of the proposed tensor product model transformation. Section 3 presents an application example including simulation results. Finally, Section 4 concludes the results.

2 Theoretical Background of the VSS

The design of a sliding-mode controller consists of three main steps. First is the design of the sliding surface, the second step is the design the control law which holds the system trajectory on the sliding surface, and the third and key step is the chattering-free implementation.

2.1 Design of the Sliding Manifold

The following linear time invariant (LTI) system is considered; first the reference signal is supposed to be constant and zero. The system (which is assumed to be controllable) is transformed to a regular form [16].

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_2 \end{bmatrix} \mathbf{u} \qquad \begin{matrix} \mathbf{x}_1 \in \Re^{n-m} \\ \mathbf{x}_2 \in \Re^{m} \\ \mathbf{u} \in \Re^{m} \end{matrix} \tag{1}$$

The switching surfaces, $\boldsymbol{\sigma}$ of the sliding mode, where the control vector components have discontinuities, can be written in the following form [17], where $\boldsymbol{\Lambda}$ is the 'surface matrix'.

$$\boldsymbol{\sigma} = \mathbf{x}_2 + \boldsymbol{\Lambda}\mathbf{x}_1 = 0 \quad \boldsymbol{\sigma} \in \Re^m \quad \text{and} \quad \boldsymbol{\Lambda} \in \Re^{mx(n-m)} \tag{2}$$

When sliding mode occurs (when $\boldsymbol{\sigma}$=0 and $\mathbf{x}_2 = -\boldsymbol{\Lambda}\mathbf{x}_1$), the design problem of the sliding surfaces can be regarded as a linear state feedback control design for the following subsystem:

$$\dot{\mathbf{x}}_1 = \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 \tag{3}$$

In (3), $\mathbf{x}_2$ can be considered as the input of the subsystem. A state feedback controller $\mathbf{x}_2 = -\boldsymbol{\Lambda}\mathbf{x}_1$ for this subsystem gives the switching surface of the whole VSS controller. In sliding mode

$$\dot{\mathbf{x}}_1 = (\mathbf{A}_{11} - \mathbf{A}_{12}\boldsymbol{\Lambda})\mathbf{x}_1 \tag{4}$$

In nineties, various linear control design methods based on state feedback are proposed for (3) to the design a stable switching surfaces in a form (4) (survey in [17]). **The main problem is that this method cannot be applied for a non-linear system which is the main challenge. The solution can be the Tensor Product model transformation.**

2.2 Sliding Surface Design based on Tensor Product Model Transformation

This section is intended to discuss the fundamentals of TP model transformation. Consider a parametrically varying dynamical system

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(z))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(z))\mathbf{u}(t) \tag{5}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

with input $\mathbf{u}(t)$, output $\mathbf{y}(t)$ and state vector $\mathbf{x}(t)$. The system matrix is a parameter-varying object, where $\mathbf{p}(z) \in \mathbf{\Omega}$ is time varying N-dimensional parameter vector, and is an element of the closed hypercube $\mathbf{\Omega} = [a_1, b_1]\mathrm{x}[a_2, b_2]\mathrm{x}\cdots\mathrm{x}[a_N, b_N] \in \Re^N$. The parameter $\mathbf{p}(z)$ can also include some elements of $\mathbf{x}(t)$.

The TP model transformation starts with the given LPV model (5) and results in the TP model representation

$$\dot{\mathbf{x}}(t) = \sum_{r=1}^{R} w^r(\mathbf{p}(z))\mathbf{A}^r\mathbf{x}(t) + \sum_{r=1}^{R} w^r(\mathbf{p}(z))\mathbf{B}^r\mathbf{u}(t) \tag{6}$$

where $w^r(\mathbf{p}(z)) \in [0,1]$ are weighting coefficients. For further details about TP model transformation, refer to [14], [15]. According to (2), a sliding surface is designed for each system $\mathbf{A}^r$ $\mathbf{B}^r$, which are assummed to be controllable.

$$\boldsymbol{\sigma}^r = \mathbf{x}_2 + \mathbf{\Lambda}^r\mathbf{x}_1 = 0 \quad \boldsymbol{\sigma}^r \in \Re^m \tag{7}$$

2.3 Control Law

There are two main approaches of design of a control law for the sliding mode on the surface. In the first ‘global’ case, to ensure that the system remains in the sliding mode ($\boldsymbol{\sigma} = 0 \quad \boldsymbol{\sigma} \in \Re^m$) the condition

$$\dot{\boldsymbol{\sigma}}\boldsymbol{\sigma}^T < 0 \tag{8}$$

should be hold. In the second ‘local’ approach, sliding mode exists only in the intersection of the switching surfaces. In this case, the condition for the existence of a sliding mode is

$$\sigma_i\dot{\sigma}_i < 0, \tag{9}$$

where the subscription i refers to the ith element of the corresponding vector. The simplest control law which can lead to ‘local type’ sliding mode is the relay:

$$u_i = M_i \cdot sign(\sigma_i) \tag{10}$$

This is easy to realize by power electronic circuits. The relay type of controller can directly control the semiconductor switching elements, but it does not ensure the existence of sliding mode for the whole state space, and relatively big values of M_i are necessary which might cause a severe chattering phenomenon. This control law is preferable if the controller's sample frequency is nearly equal to the maximum switching frequency of semiconductor switching elements.

If sliding mode exists then there is continuous control, so-called ‘equivalent’ control, $\mathbf{u}_{\mathbf{eq}}$, which can hold the system on the sliding manifold. In the practice, there is never perfect knowledge of the whole system and its parameters. Only $\hat{\mathbf{u}}_{\mathbf{eq}}$, the estimation of $\mathbf{u}_{\mathbf{eq}}$, can be calculated. Since $\mathbf{u}_{\mathbf{eq}}$ does not guarantee the convergence to the switching manifold in general, a discontinuous term is usually added to $\hat{\mathbf{u}}_{\mathbf{eq}}$.

$$u_i = \hat{u}_{eq,i} + M_i \cdot sign(\sigma_i) \tag{11}$$

The control laws (11) do not control the semiconductor switching elements directly; additional PWM is needed. Usually, this is no problem since the switching frequency of the semiconductor elements can be much higher then the sampling frequency of the fastest digital controller. The general concept of TP model based control strategies is that the control signal is the weighed sum of the control signal of the component systems

$$\mathbf{u} = \sum_{r=1}^{R} w^r(\mathbf{p}(z))\mathbf{u}^r \tag{12}$$

2.4 Chattering Free Implementation

The chattering is essential in the basic VSC due to the requirement that the system state must stick to the switching surface. Obviously this requirement is too restrict when only finite switching rate is available. Replacing the switching surface to the sliding sector may enable the system state to move continuously. From now on single input and two sliding surfaces are assumed, the whole state space is divided into three regions

$$\begin{aligned} &R_1 \in \{\mathbf{x} \mid \sigma^1(\mathbf{x}) < 0 \quad \text{and} \quad \sigma^2(\mathbf{x}) < 0\} \\ &R_2 \in \{\mathbf{x} \mid \sigma^2(\mathbf{x}) > 0 \quad \text{and} \quad \sigma^2(\mathbf{x}) > 0\} \\ &R_3 \in \{\mathbf{x} \mid \sigma^1(\mathbf{x})\sigma^2(\mathbf{x}) \le 0\} \end{aligned} \tag{13}$$

Here the region R_3 is a sliding sector introduced in [18]. The control is composed of two terms

$$u = u_c + u_d \tag{14}$$

where u_c is a feedforward compensation term based on the estimation of the 'equivalent' control and the given uncertainty bounds, u_d is a switching term to suppress the system parameter variations and disturbances. According to [18], in conventional (non TP) case

$$u_c = \hat{u}_{eq} \text{ and } u_d = \begin{cases} M\operatorname{sign}\left(\dfrac{\sigma^1+\sigma^2}{2}\right) & \text{if} \quad \mathbf{x} \in R_1 \cup R_2 \\ M\left(\dfrac{\sigma^1+\sigma^2}{\left|\sigma^1\right|+\left|\sigma^2\right|}\right) & \text{if} \quad \mathbf{x} \in R_3 \end{cases} \tag{15}$$

The sliding sector control inherits the robustness of the classical sliding mode control in certain cases (see details in [18]). The sliding sector concept can be extended to the TP model based sliding mode control as well.

$$u_c = \sum_{r=1}^{2} w^r((p(z))\hat{u}_{eq}^r \text{ and } u_d = \begin{cases} M\operatorname{sign}\left(\dfrac{\sigma^1+\sigma^2}{2}\right) & \text{if} \quad \mathbf{x} \in R_1 \cup R_2 \\ M\left(\dfrac{\sum_{r=1}^{2} w^r((p(z))\sigma^r}{\sum_{r=1}^{2} w^r((p(z))\left|\sigma^r\right|}\right) & \text{if} \quad \mathbf{x} \in R_3 \end{cases} \tag{16}$$

3 Application

The experimental system consists of a conventional DC servo gear motor with encoder feedback and variable inertia load coupled by a relatively rigid shaft, as shown in Fig 1. The controller is implemented using a DSP as the computation engine.

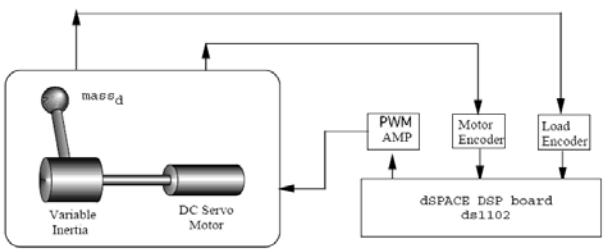

Figure 1
The experimental system

3.1 System Equations

In the course of control design, the flexibility of the shaft is ignored. The state variables are the shaft position, θ, the shaft angular velocity, ω, and the input current, i, the control signal u is the motor voltage.

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{i} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{K_t}{J} \\ 0 & -\frac{K_\omega}{L_a} & -\frac{R_a}{L_a} \end{pmatrix} \begin{pmatrix} \theta \\ \omega \\ i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{pmatrix} u \tag{17}$$

where J is the inertia of the motion control system, K_t and K_ω are the torque constant and the back-EMF constant R_a and L_a are the resistance inductance of the armature. The effect of $mass_d$ is considered as a disturbance. The model calculated from the nominal parameters of the system is as follows:

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{i} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 42 \\ 0 & -4600 & -2450 \end{pmatrix} \begin{pmatrix} \theta \\ \omega \\ i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1100 \end{pmatrix} u \tag{18}$$

The viscous, Coulomb and Stribeck frictions are modelled by Hess and Soom [19] in the following way, where the second two terms are nonlinear:

$$\dot{\omega} = -\underbrace{\frac{F_v}{J}\omega}_{\text{viscous term}} - \underbrace{\left(\frac{2F_c}{J(1+e^{-500\omega})} - \frac{F_c}{J}\right)}_{\text{Coulomb term}} - \underbrace{\left(\frac{\frac{2F_s}{1+e^{-500\omega}} - F_s}{J(1+(v/v_s)^2)}\right)}_{\text{Stribeck term}} + \frac{K_t}{J} i \tag{19}$$

where F_v, F_c and F_s are constants for the viscous, Coulomb and Stribeck frictions, v_s is the characteristic velocity of the Stribeck curve. F_v was given in data sheet of the servo motor, F_c, F_s and v_s are selected after some tests. Fig. 2 shows the simulated Stribeck curve.

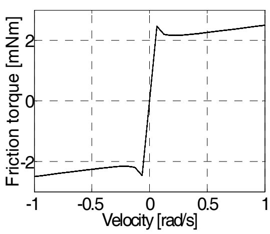

Figure 2

Stribeck curve by simulation

Applying the tensor product transformation, the above nonlinear system can be modelled by the combination of the following two linear systems.

$$\mathbf{A}^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -20.2 & 42 \\ 0 & -4600 & -2450 \end{pmatrix} \mathbf{B}^1 = \begin{pmatrix} 0 \\ 0 \\ 1100 \end{pmatrix} \tag{20}$$

$$\mathbf{A}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -6239 & 42 \\ 0 & -4600 & -2450 \end{pmatrix} \mathbf{B}^2 = \begin{pmatrix} 0 \\ 0 \\ 1100 \end{pmatrix} \tag{21}$$

The two weighing coefficients as functions of the speed are shown in Fig. 3.

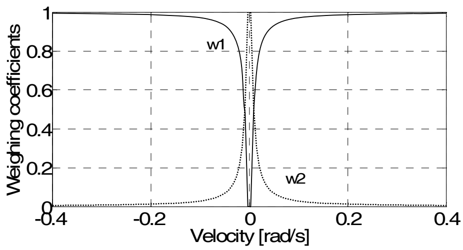

Figure 3
The weighing coefficients as a function of the velocity

It is easy to explain. The nonlinear friction terms are modelled by varying viscous coefficient, which is represented by the a_{22} element in the system matrix. $\mathbf{A}^1$ with small viscous coefficient dominates at the high speed range, where the Coulomb friction is relatively small and the $\mathbf{A}^2$ with very big viscous coefficient dominates at the low speed range, where the Coulomb friction is relatively big.

3.2 Sliding Surface Design

According to (7) and (17), the surfaces have the following form:

$$\sigma^r = i + \lambda_\omega^r \omega + \lambda_\theta^r \theta = 0 \quad \text{where} \quad r{=}1{,}2 \tag{22}$$

The poles for the reduce order systems of

$$\mathbf{A}_r^1 = \begin{pmatrix} 0 & 1 \\ 0 & -20.2 \end{pmatrix} \quad \mathbf{B}_r^1 = \begin{pmatrix} 0 \\ 42 \end{pmatrix} \tag{23}$$

$$\mathbf{A}_r^1 = \begin{pmatrix} 0 & 1 \\ 0 & -6239 \end{pmatrix} \quad \mathbf{B}_r^2 = \begin{pmatrix} 0 \\ 42 \end{pmatrix} \tag{24}$$

are selected as

P=[-17 -35]. (25)

Applying the Matlab pole placement function:

$$\lambda_\theta^1 = \lambda_\theta^2 = 14.1667 \quad \lambda_\omega^1 = 0.7571 \quad \lambda_\omega^2 = \text{-}147.3 \tag{26}$$

3.3 Simulation Results

As model verification, the real and simulated velocities (ω_r, ω_s) are compared in Fig. 4, where the input voltage of the motor is a shifted sinusoidal function with amplitude of 12 V (open loop responses). The value of the input voltage is divided by 5 to plot the velocity and input voltage in the same figure. One kind of nonlinearity of the system is borne from the huge friction of the harmonic gear. It can be seen in the Fig. 4, if the motor is in standstill, at least 2 V should be switched across the motor to start it. On the other hand, the motor sticks, if the input voltage is under 1.2 V. According to Fig. 4, the simulated model is acceptable from engineering point of view. The power electronic PWM unit is saturated at 22V. It is also a kind of nonlinearity which can be handled by TP model but this paper concentrates on the friction that is why only the nonlinearity of the friction is simulated by TP model.

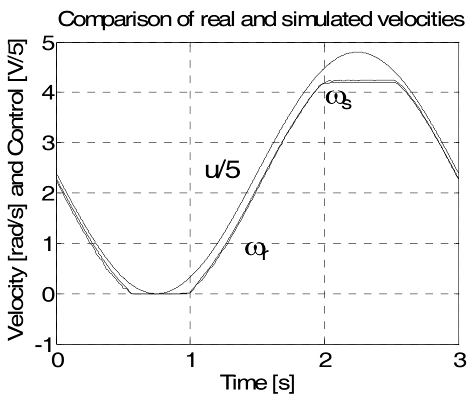

Figure 4
The open loop responses for sinusoidal input voltage

Performances of two controllers are compared. In both cases, the system starts from the following initial state

θ=1 rad, ω=0 rad/sec and i=0 A (27)

The aim of the controller is to move all state variables to 0. The sampling frequency of both controllers is relatively small, 100 Hz.

CONTROLLER C-SMC

It is a classical sliding mode controller (C-SMC), where the sliding surface σ^1 and (11) type control law are selected. The equivalent control is calculated from ($\mathbf{A}^1$ $\mathbf{B}^1$) system matrixes.

CONTROLLER TP-SMC

It is a TP model based sector sliding mode controller(TP-SMC), where the two sliding surfaces are selected by (26) and (16) type control law is applied with two linear system components (20),(21) and the weighting coefficients of Fig. 3. Two equivalent controls are calculated from ($\mathbf{A}^1$ $\mathbf{B}^1$) and ($\mathbf{A}^2$ $\mathbf{B}^2$) system matrixes.

There is a small difference between two position responses in Fig. 5 since the conventional (static) sliding surface cannot be identical to the sliding sector. The main difference appears in the control activities and in the velocity responses (see in Figs. 6 and 7). The conventional sliding mode is very robust but it needs intensive control action (see in Fig. 6), which might cause significant audio noise as well. The chattering of the velocity could be reduced by increasing the sampling frequency but this paper demonstrates that the reduction of chattering and the intensity of the control action (the audio noise) is significant at the same sampling rate, if the TP based sector sliding mode is applied instead of the traditional sliding mode control.

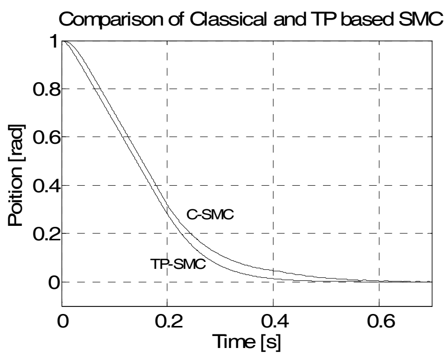

Figure 5
Comparison of the position responses

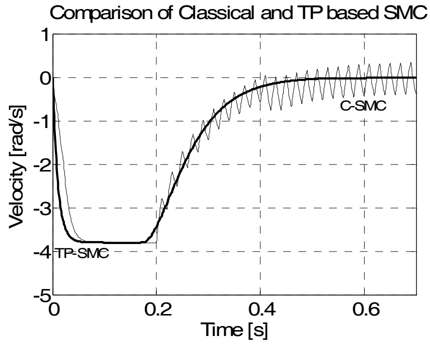

Figure 6

Comparison of the velocity responses

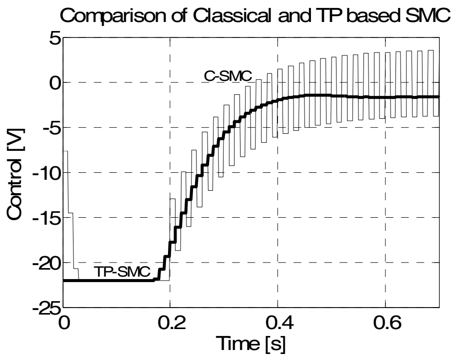

Figure 7

Comparison of the control signals

Conclusions

In this paper, a modified variable structure control strategy with continuous switching control has been developed in detail for the nonlinear system with uncertainty. The control strategy can be regarded as the extension of conventional VSS based sliding mode control method through expanding the switching surface to the sliding sector. The sliding sector is designed by a tensor product model transformation. The major advantage of the proposed control scheme is the introduction of the continuous switching control which successfully achieves smooth control response and retains the robustness of VSC simultaneously. Both theoretical analysis and simulations demonstrate the attractiveness and the asymptotic stability of the sliding sector with the use of the proposed switching control which is essentially an interpolated control.

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