How Safe the Human-Robot Coexistence Is? 
Theoretical Presentation

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Abstract: It is evident that industrial robots are able to generate forces high enough to injure a human. To prevent this, robots have to work within a restricted space that includes the entire region reachable by any part of the robot. However, more and more robot applications require human intervention due to superior abilities for some tasks performance. In this paper we introduce danger/safety indices which indicate a level of the risk during interaction with robots, which are based on a robot’s critical characteristics and on a human’s physical and mental constrains. Collision model for a 1 DOF robot and “human” was developed. Case study with further simulations was provided for the PUMA 560 robot.

Keywords: robotics, safety, human-robot interaction, danger index

1 Introduction

Most safety standards require an installation of the safeguarding systems, so any access to the hazard (robot work space) is prevented, or the cause of hazard is removed without requiring specific conscious action by the person. The prescribed action to be taken by the robot system upon detecting an intrusion into the safeguarding space is to remove all drive power and all other energy sources. Thus, robots must be surrounded by the safeguarding space and production must be designed to allow the maximum number of tasks to be performed with personnel outside the safeguarding space. However, this approach is not applicable for the new tendency in robotics where humans and robots interact in unstructured space and where their working zones are overlapped (social robotics, collaborative tasks, etc.) [1-3]. A typical situation in industrial robotics where a human operator can be hit, trapped between the safety equipments and the robot parts is during maintenance, teaching or collaboration [4], [5]. To avoid or minimize the severity of injury we should keep the risk level at a minimum.
Concerning injuries caused by robots, only very little data or literature is available. In [6] the United Auto Workers (UAW) union published a report which provides raw data on various injuries related to robot operations. There are many types of injuries which could potentially occur during the interaction between a human and a manipulator. These include cuts or abrasions, which might result from contact with a sharp or abrasive surface, as well as more serious injuries including bone fracture which could result from manipulator pinch points or direct crush loads. However, when a human operator works near a robot, the most dangerous accident is the potential impact with large loads that may cause serious injury or even death.

Therefore, the danger criterion should be constructed from measures that contribute to reducing the impact force in case of unexpected human-robot impact, as well as reducing the likelihood of the impact itself. Concerning to this issue we introduced a new criteria (danger index) which is based on the critical measures of impact forces, accelerations and distances to reduce a probability of the dangerous collision.

2 Related Work

A number of standard indices of injury severity have been developed. Some of them attempt to relate resulting head acceleration to the severity and likelihood of injury [7], [8], [9], [10]. The basis of these measures is the Wayne State University Tolerance Curve (WSTC) (See Fig. 1) which relates acceleration and duration to the likelihood of severe brain injury.

To evaluate the potential for serious injury due to impact an empirical formula was developed by the automotive industry to correlate head acceleration to injury severity known as the Head Injury Criteria (HIC) [11], which is computed as the
maximum integral of the resultant acceleration of the centre of mass of the head during the crash (1).

\[
HIC = \Delta t \left( \frac{1}{\Delta t} \int_{t_1}^{t_2} a_h dt \right)^{2.5}, \Delta t = t_2 - t_1
\]  

(1)

Where \(a_h\) is the resulting acceleration of the human head and \(\Delta t\) is a period of impact that should not be more than 15 ms.

Prasad and Mertz [12] introduced a set of curves which statistically relates measured HIC values to the severity and likelihood of a head injury. Using these curves, in combination with evaluated HIC values, it is possible to define the level of an injury resulting from a given head acceleration time history. The resulting injury indices can be also used to judge the severity of the injury with further consultation of biomechanical expertise, like e.g. the so called Abbreviated Injury Scale (AIS) [13]. Figure 2 illustrates an exponential correlation between AIS and HIC criterion, which evaluation is based on post mortal experiments. It is seen that from a certain value of AIS (1.6) the HIC rises drastically. The HIC of 250 is correlated to the AIS1+ value where injuries to the human are negligible.

The HIC is a commonly used frontal impact criteria that has been used for decades to assess the level of head injury risk in frontal collisions. A HIC of 1000 is conventionally considered to represent the threshold where linear skull fractures normally begin to appear. According to this assumption a head can sustain acceleration more than 90 g. However, for the lateral or transversal impacts this value can result in severe injuries especially if this acceleration was caused by collision with a rigid surface (manipulator arm). Therefore, more experiments have to be provided and more restricted boundaries have to be introduced.

![Figure 2](image)

The AIS – HIC dependency curve [13]

Most research related to the HIC criteria were based on the automobile crash-testing results defined this criterion as an impact involving a collision of the head with another solid object at appreciable velocity. This situation is generally characterized by large linear accelerations and small angular accelerations during the impact phase.
In the work [14] Head Injury Criterion was evaluated for robot masses up to 500 kg with the linear velocities (0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3) m/s. Results indicated that at some point increasing robot mass does not result in a higher HIC. Moreover, according to this criterion, no robot, whatever mass it had, became dangerous at the operating speed up to 2 m/s as long as the time of impact was less than 36 ms. In this research typical severity indices, established in the automobile industry, were claimed to be not applicable for a human-robot interaction domain in view of the much lower operating velocities.

In another study [15] Manipulator safety index (MSI) was introduced, which evaluation was also based on the HIC criterion, where a head acceleration was computed from a human robot a collision model. The resulted index depends on the manipulator's effective inertia, interface stiffness, and initial robot velocity mappings. However, in graphical representations of this analysis the velocity and the stiffness characteristics become dominant while the inertial weight - negligible. Moreover, in view of the conflict of some parameters (low stiffness with high inertia configuration) this index can not reflect the real hazard caused by the robot under its certain configurations. In spite of the fact that the advantage of the HIC criteria application in the robotic field is questionable at some extent in this research evaluations were also based on this criteria.

3 An Introduction to a Danger Severity Evaluation

Since it is not feasible to adequately treat all different contact types of injuries in this work only blunt contacts were considered. To develop a quantitative measure which relates the severity and likelihood of injury to the physical characteristics of a given manipulator factors as force, acceleration and distance were taken into account. In the case of mechanical injury at a collision accident, the severity of an injury mostly depends on impact force and the likelihood depends on the distance to impact area before collision. In turn, an impact force mostly depends on robots physical characteristics, specific configurations, approaching speed, direction, and the contact duration. [16], [17] Among the minor factors that contribute to the Index are diverse robot tasks, failure rates, presence of any safety features, shape of the instrument, joint compliance, control methods, etc. Moreover, the severity of impact will also depend on the human factor [18]. For instance, characteristics as age, sex, weight will change personnel physical and mental hazard perception as well as a reaction on it. In this paper human physical constrains were considered to establish the boundaries on the robot performance, assuming that a physical contact may occur. Critical characteristics were obtained from the biomechanical injury/pain tolerance estimations, acquired experimentally in the works [7], [19]. These results were approximated and the mean values were used.
The proposed generalized form of the danger index consists of a linear combination of qualities that take into account the relevant distance \( (D_{id}(t)) \), the contact force \( (D_{if}(t)) \) and the human head acceleration factors \( (D_{ia}(t)) \) (2). The sum of these indices with their corresponding weights has to be less or equal to one for a safe human robot interaction:

\[
DI = \alpha_d D_{id}(t) + \alpha_f D_{if}(t) + \alpha_a D_{ia}(t) \leq 1
\]

\[
D_{id}(t) = \frac{L_c}{L_i}, \quad D_{if}(t) = \frac{F_i}{F_c}, \quad D_{ia}(t) = \frac{a_i}{a_c}
\]  

(2)

Where \( F_i \) is an actual value of the force exerted by a manipulator, i.e. a producible impact force of the robot, \( F_c \) is a critical, admissible force, that doesn’t cause serious injury to a human at a collision body part. \( L_i \) is an actual distance measured from the visual or sensory monitoring system, \( L_c \) is a distance that robot overpass after stopping signal was sent to the robot control. This stopping distance mainly depends on the actual robot speed \( v_i \) and its load. Parameter \( a_i \) is an acceleration of a head measured after collision with manipulator that is compared with a critical one \( a_c \) obtained from the AIS scaling.

All indices are time dependent. An acceleration related index is examined under the condition when the head acceleration (or other body part) achieves the maximum value. This occurs at the minimum or critical time interval \( \Delta t \). [20] Coefficients \( \alpha_d \), \( \alpha_f \) and \( \alpha_a \) are weights of the distance, force and acceleration terms respectively. The indices evaluation and the corresponding weights distribution is based on the initial task description, risk assessment results and information available during analysis. For instance, for collaborative tasks in close vicinity distance factor is not important since the distance is negligible or even contact between human and robot is possible. However, if a robot effective mass and, as a consequence, exerted force at some configurations is greater than the admissible value human can be injured. In this case the force related danger index plays the dominant role. On the other hand, if a robot is performing task in the automatic regime with the maximum (optimal for the task) characteristics, it is essential to keep the safe distance to avoid the likelihood of impact under these conditions, i.e. the weight for the distance related index will be under consideration.

### 3.1 Distance Related Danger Index

Sufficient distance provides with time to reduce impact force by braking actions to avert the collision. Thereby, keeping that distance can be a criterion for a danger evaluation. To compute this value we should know mutual robot, personnel approaching speed and the time needed to stop all movements.

A minimum distance to hazard \( L_c \) (3) depends on a robot’s operational speed, a sensory system reaction time, control system response time and robot’s braking characteristics. Time \( T_i \) expresses a robot’s stopping time that varies depending on
the applied drivers stopping category, braking system idle time and the safety system response time (if the safety distance is controlled by the external present sensing device). [21] It was assumed that in (3), (4) acceleration (deceleration) is a constant value.

\[ L_c = v_i \times T_i \]  
\[ L_i = (v_i + v_h) t - a t^2 / 2 > L_c \]  
\[ D_{i_t} = (v_i \times T_i) / ((v_i + v_h) t - a t^2 / 2) < 1 \]

When the distance to contact is sufficient (4) we have time to decelerate the robot and avoid the undesired impact. When the speed of a robot can be reduced with some deceleration to the condition when the contact with a human becomes not likely, the distance is claimed to be safe. At this distance robot can move at its normal operational speed and in the case of a safety distance violations, it decelerates or cease all movements. In (4), (5) \( v_h \) is a human average walking, hazard approaching speed. According to the human factor analysis its mean value is 1.6 m/s. \( v_i \) is a robot operational speed, \( t \) is a time scale. At the distance \( L_i \) robot is fulfilling its task at the max speed or at the speed needed for the effective task performance till there is no human entering the monitoring area. As soon as a no authorized access to this zone has been recognized robot’s speed is decreases with an acceleration \( a \). If the critical distance \( L_c \) is overrun (or near to be), a robot is forced to stop. This situation occurs if human continues to move toward the robot in spite of the warnings, or if the robot does not have enough time to decelerate to the speed established as a “safe” for the current distance at the time \( t \).

Therefore, the distance related danger index is evaluated basing on the relation between the critical and the current distances, where the later should be kept always greater than the critical one to avoid undesirable contact. This danger index formulation is represented in (5). Danger index can be displayed as a circle (sphere) which radii corresponds to the value 1. All characteristics inside this circle will comply with the safety requirements, exceeded values will require appropriate danger reduction procedures.

For our analysis robot speeds \( v_i \) were set in the interval \([0, 0.2, 0.7, 1, 1.5, 2, 2.9]\) m/s. Time \( T_i \) with respect to the speed \( v_i \) was chosen according to the experimental results provided in the work [22] From the relation in (5) we define the time interval where in compliance with the danger index analysis this function should be less than zero (6). Fulfillment of this condition decreases the probability of the human robot contact since the minimum distance between them is provided by the danger index control.

\[ f(t) = at^2 / 2 - (v_i + v_h) t + v_u T_i < 0 \]  

From Fig. 3 we can see that the requirements are met within the time interval \([t_1, t_2]\). Graphical representations in Fig. 4 (a) indicate the minimum required
deceleration values for the speed range: 0.25, 0.6 and 1 m/s. It is evident that lower velocities need less time to decelerate. Integrating human walking speed in the danger index formulation system has to apply greater accelerations to convey with the safety (danger index) requirements. (See Fig. 4 (b))
3.2 Force Related Danger Index

The force $F$ as well as acceleration $a$ in general can be defined as a function of different influencing elements, i.e.: approaching velocity, robot effective mass, inertia, stiffness, kinetic energy, etc. In this study an effect of the manipulator arm effective mass and approaching speed is investigated. According to the Newton’s theory the impact force depends on the robot (here) acceleration or speed at the moment of collision, therefore, both characteristics will be considered in the analysis. In general, the second Newton's low formalization provides with description of the linear motion, where applied force $F$ depends on the mass $m$ moving with acceleration $a$. This formulation can be also described in terms of the linear momentum $mv$ where a rate of change of the linear momentum is equal to the applied forces. With a reference to a collision model (will be discussed later) we yield (8):

$$\frac{m \nu_0 - m \nu'}{\Delta t} = F_u$$

(8)

Where $\Delta t$ is a time of the collision duration, $\nu'$ is a velocity after impact, $m_u$ is a scalar value of the mass at the direction $u$, $F_u$ is a resulted force at the same direction. At some conditions the value of this force can become infinite or very large. This situation is very dangerous especially when the human is under the risk of impact. Therefore, to establish tolerable boundaries on the exerted force magnitudes is an issue that has to be investigated. One of the possible solutions is to introduce a danger index (9) that would indicate an admissible level of the controlled parameter.

$$D_i = \frac{F_i}{F_c} \leq 1 \quad \left( D_i = \frac{a_i}{a_c} \leq 1 \right)$$

(9)

In the force related danger evaluation the producible force $F_i$ should be is compared with the critical one $F_c$, which is maximum “safe” value, and established basing on the largest force magnitude that does not cause serious injury or pain to a human (here). In the course of the injury limit evaluation the most vulnerable part of a human body was considered, head.

Similarly, with respect to Newton’s law, acceleration related index can be yielded. However, it was also decided to investigate HIC criteria and its applicability for a HRI field, thus, two critical acceleration will be used for a acceleration related danger estimation, based on Newton’s low and on HIC index.

According to the studies provided in [22] the level of injury can be measured on the basis of the head human skull bone fractures, however, the threshold of the fracture highly depends on the contact area. For instance, the fracture force of the occipital bone is estimated 6.41 KN, while the fracture force of the maxilla bone was measured of only 0.66 KN. On the other hand, considering that analysis is provided for friendly human-robot interactions even any causes of pain should be
avoided. Since this characteristic also depends on the area of impact, each body part can be considered separately.

Therefore, it was determined for the most critical estimations (pain tolerance) apply more restrictive tolerance limits on the robot exerted forces. There have been few reports discussing the human pain tolerance limits when static or dynamic stimuli are applied to the whole body. For this study a critical force value causing pain was derived from the analysis provided in the work [19], where somatic pain tolerance is investigated. Parameters of the pain tolerance were acquired from a human response on the applied mechanical stimuli. For instance, for the parts under the most frequent exposure to the hazard (hand, arm, back and head) the critical force was found as 140 N, 180 N, 240 N and 130 N respectively. In this analysis, for the further evaluations an effect of a head impact will be investigated. Thus, the most restrictive danger criteria will be based on the force equal 130 N.

4 Manipulator Effective Mass Formulation

For a multi-link manipulator the effective mass at the direction of impact is changing with each robot configuration. We consider the impact itself in the operational space of a manipulator, therefore, the mass and inertial properties have to be evaluated in that space. Since the mass properties of a manipulator are generally expressed with respect to its motion in joint space the transformation method should be introduced.

The manipulator’s dynamic model in the joint (10) and operational spaces (11) is described in [23].

\[
M(q) \ddot{q} + v(q, \dot{q}) + g(q) = \tau
\]  
\[
M_\epsilon \ddot{x} + v_\epsilon(x, \dot{x}) + g_\epsilon(x) = F
\]  

Here \( M(q) \) is \( n \times n \) joint and \( M_\epsilon \) is end effector kinetic energy matrices, \( v(q, \dot{q}) , v_\epsilon(x, \dot{x}) \) are the vectors of centrifugal and cariokes forces, \( g(q) \) is the vector of gravity, \( \tau \, , F \) are the generalized vectors of joint and end effector force respectively. The relation between two matrices can be expressed as in (12).

\[
M_\epsilon(q) = (J(q)M^{-1}(q)J^T(q))^{-1}
\]  

Where \( J(q) \) is the basic Jacobian associated with the end-effector linear and angular velocities and \( M(q) \) is a symmetric positive defined mass matrix. Assuming that impact occurs within a robot’s transition movement (close distance collision), \( J(q) \) is equal to \( J_\epsilon(q) \) (Jacobian matrix associated with the linear velocity of the end effector). If an impact occurs when the end-effector is moving along an
arbitrary direction, a kinetic energy matrix in this case is a scalar \((m)\) representing
the mass perceived at the end effector (point of impact) in response to the
application of a force \((F)\) along this direction (13) (See Fig. 5).

\[
\frac{1}{m_v} = J^{T} v_{v}(q)M_{v}^{-1}J_{v}(q); \quad J_{v}(q) = u^{T}J_{v}(q)
\]

To evaluate the effective mass at the direction of impact the mass matrix \(M_{v}(q)\)
should be diagonalized in order to avoid the effect of coupling between its
elements. One of the methods that can be introduced is the eigenvectors \((V)\) and
eigenvalues \((\lambda)\) determination with an ellipsoidal geometrical representation of
the mass matrix properties as it is shown in (14). This representation provides a
description of the square roots of the effective mass properties (eigenvalues) in the
arbitrary directions (eigenvectors) [24].

\[
\left(\frac{x}{\sqrt{\lambda_1}}\right)^2 + \left(\frac{y}{\sqrt{\lambda_2}}\right)^2 + \left(\frac{z}{\sqrt{\lambda_3}}\right)^2 = 1
\]

The eigenvalues and eigenvectors associated with the matrix \(M_{v}(q)\) or its inverse
provide a useful characterization of the bounds on the magnitude of the mass
properties. The eigenvectors of this matrix define the principal directions of the
ellipsoid and the inverse of the square roots of the eigenvalues indicate the
corresponding equatorial radii. Moreover, by identification of the maximum
eigenvalues (eigen vectors) characteristics (15), it is possible to assess the extent of
the manipulator actual configuration danger and establish corresponding
boundaries in compliance with safety requirements and danger criteria of the task
(16). (See Fig. 5)

\[
\frac{1}{m_{\text{max}}(\lambda_{\text{max}})} = V^{T}(\lambda_{\text{max}})M_{v}^{-1}(q)V(\lambda_{\text{max}})
\]

\[
\frac{1}{m_{c}(Di)} = u_{c}^{T}(Di)M_{v}^{-1}(q)u_{c}(Di)
\]

Figure 5
Effective mass ellipsoid with two intersecting danger index (Di1, Di2) spheres
5 Collision Modeling

For more precise danger indices analysis we refer to the dynamic simulation of the impact which is based on the one DOF mass-spring collision model (See Fig. 6). An assumed dynamic model is described in the equations of motion in (17):

\[ M_r a_r + K_e (x_r(t) - x_h(t)) = 0 \]  

(17)

In here \( M_r \) and \( a_r \) are manipulator arm effective mass acting in the direction of impact and its deceleration value after collision respectively, \( K_e \) is a measured effective stiffness, difference in displacements \( x_r \) and \( x_h \) describes a robot and a human (head) mutual allocation after impact.

In this assumption a mass \( M \) of the manipulator is an effective mass that reflects the inertial manipulator properties at the point of impact. The real value of the acceleration \( a_h \) and the period of impact can be found from the relations below, assuming that the impact occurs with a maximum spring compression \( x_{\text{max}}(t) \) defined from (18):

\[ \frac{1}{2} M v^2_{\text{or}} = \frac{1}{2} K_e x^2(t)_{\text{max}} \]

(18)

\[ x(t)_{\text{max}} = \sqrt{\frac{M v^2_{\text{or}}}{K_e}} \]

(19)

\[ x(t) = x(t)_{\text{max}} \cos(\omega_n t) \]

Setting first derivative of the time dependant generalized form equation of motion (19) equal to 0 the impact period when a head is exposed to a maximum acceleration \( a_h \) can be evaluated:

\[ \dot{x}(t) = x(t)_{\text{max}} \sin(\omega_n t) = 0 \Rightarrow t(a_{\text{max}}) = \frac{\pi}{\omega_n} = \frac{T}{2} \]  

(20)
Here $\omega_n$ is a natural frequency of the oscillation after impact. For stiff surfaces, as a robot is, this period is assumed to be less than duration of the impact ($T/2 \leq \Delta t$; $\Delta t = 0.015 ms$). Further, considering provided above measures, a manipulator and a head accelerations after impact had been estimated:

$$a_x = \frac{M}{m} \omega_n \omega_n \cos(\omega_n t)$$

$$a_x = \frac{M}{m} \omega_n \omega_n \cos(\omega_n t)$$

(21)

(22)

Where $\cos(\omega_n t) = 1$ if $-T/2 < t < T/2$

Accelerations can be also computed for the mass-spring-damper system, which behavior depends on the natural damping ratio $\xi_n$ (23). The system is critically damped when $\xi_n = 1$, over damped if $\xi_n > 1$, and oscillatory damped when $\xi_n < 1$. The equation of motion for this system is shown in (24):

$$x(t) = x(t)_{\text{max}} e^{-\xi_n t/2} \cdot \cos(\omega_d t)$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$T/2 = \frac{\pi}{\omega_d \sqrt{1 - \xi^2}}$$

(24)

Here $\omega_n$ is a damped natural frequency, $\xi$ is a damping ratio and $C$ is a friction coefficient.

Consequently, the head acceleration can be found similarly to (22) and expressed as in (25):

$$a_x = \frac{M}{m} \omega_n \omega_n (\xi^2 - 1) \cos(\omega_d t)$$

(25)

Where $\cos(\omega_d t) = 1$ if $-T/2 < t < T/2$

However, in a view of the fact that the robot (here) has a very high stiffness, damping ratio will be very small ($10^{-4}$) and doesn’t contribute significantly to a danger index value. Therefore, for the further computations mass spring damping system will not be considered.

Finally, according to estimations provided in (2), (9), knowing acceleration and force critical values we can establish the acceleration and the force related danger indices.
6 HIC-based Danger Estimation

According to AIS scale a head can sustain quite high accelerations if the loading is relatively short and if the time duration is relatively long.

Table 1 [8] demonstrates a relation between the peak linear head acceleration and the severity injury level. In this research we consider situations where “no” or only “minor” injuries are acceptable. Therefore, according to the AIS scale the threshold for a maximum head acceleration has been established up to 50 g with the assumed impact duration $\Delta t=15$ ms. These assumptions imply the HIC 265 computed in (26) that is correlated with the AIS1 level.

$$HIC = \Delta t \left[ \frac{\Delta V}{g \Delta t} \right]^{2.5} = 0.015 \times [50]^{2.5}$$  \hspace{1cm} (28)

<table>
<thead>
<tr>
<th>Peak linear acceleration, g</th>
<th>AIS head injury severity</th>
<th>Injury interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;50</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>50-100</td>
<td>1</td>
<td>Minor</td>
</tr>
<tr>
<td>100-150</td>
<td>2</td>
<td>Moderate</td>
</tr>
<tr>
<td>150-200</td>
<td>3</td>
<td>Serious</td>
</tr>
<tr>
<td>200-250</td>
<td>4</td>
<td>Severe</td>
</tr>
<tr>
<td>250-300</td>
<td>5</td>
<td>Critical</td>
</tr>
<tr>
<td>&gt;300</td>
<td>6</td>
<td>Unsurvivable</td>
</tr>
</tbody>
</table>

By substituting identified accelerations in Ch5 into a HIC criterion formulation (1) we can establish a relation between the AIS scale and the manipulator based collision model as in (29):

$$HIC = \Delta t \left[ \frac{1}{g \Delta t} \int_{t/2}^{t/2} \frac{M}{m} \sqrt{\frac{M}{M + m} \cos(\omega_0 t) dt} \right]^{2.5}$$  \hspace{1cm} (29)
Taking a definite integral from (29), a new HIC criteria that depends on the manipulator’s operating characteristics can be yielded as following:

\[
HIC = \Delta t \left[ \frac{2v_0}{m} \sqrt{\frac{M}{m+m}} \sin\left(\frac{\Delta t}{2}\right) \right]^{2.5} 
\]

(30)

Where \( \sin\left(\omega_n \frac{\Delta t}{2}\right) = 1 \) for \( \Delta t > \frac{T}{2} \)

Hence, human head acceleration from HIC index is:

\[
a_{HIC} = \frac{2v_0}{m} \sqrt{\frac{M}{m+m}} \frac{\Delta t}{\Delta t} 
\]

(31)

Furthermore, substituting this expression into (9) a HIC-based acceleration and force related danger criterion can be obtained:

\[
D_{a_HIC} = a_{HIC} = \frac{2v_0}{m} \sqrt{\frac{M}{m+m}} a_c \leq 1
\]

(32)

\[
D_{f_HIC} = \frac{f}{f_c} = \left( \frac{2v_0}{m} \sqrt{\frac{M}{m+m}} \right) / f_c \leq 1
\]

(33)

By definition, these two danger evaluations should be equivalent and both can be used independently for the safety level identification.

7 Case Study

To cite an example a PUMA 560 robot was applied (See Fig. 7). First, an effective mass matrix at a given robot configurations: \( q_1(0), q_2(0), q_3(0), q_4(0), q_5(90), q_6(10) \) grad was computed. Analysis was provided with two assumptions: the last 3 joints of the robot do not contribute significantly into a kinetic energy matrix of the PUMA robot, therefore, the mass matrix \( M(q) \) in joint space has a dimension 3x3; the distance before collision is relatively small, thus, the motion of the end effector in the direction of impact is considered translational.
In Fig. 7 vectors P3 and P6 identify the center of mass of the link 3 and the point of impact respectively:

\[
P_{32} = \begin{bmatrix} C_1(a_2C_2 + a_1C_{22}) - d_2S_2 \\ S_1(a_2C_2 + a_1C_{22}) + d_2C_1 \\ -a_1S_{22} - a_2S_2 \end{bmatrix}
\]

\[
P_{62} = \begin{bmatrix} C_1(d_2(C_2S_2 + S_2C_1) + S_2d_4 + a_1C_{23} + a_2C_2) - S_1(d_4S_2S_6 + d_2) \\ S_1(d_2(C_2S_2 + S_2C_1) + S_2d_4 + a_1C_{23} + a_2C_2) + C_1(d_4S_2S_6 + d_2) \\ d_6(C_2S_6 + S_2C_3S_1) + C_2d_4 - a_1S_{22} - a_2S_2 \end{bmatrix}
\]

According to (10) and basing on the evaluations provided in [25] the mass matrix in a joint space was computed according to the assumption in (35) and for the considered joint angles its final numerical form is presented in (36):

\[
M(q) = m_1J_{11}^1J_{11}^1 + m_2J_{12}^1J_{12}^1 + m_3J_{13}^1J_{13}^1 + J_{22}^2J_{22}^2 + J_{23}^2J_{23}^2 + J_{33}^2J_{33}^2
\]

\[
M(q)^{123} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{22} & m_{22} & m_{23} \\ X & X & m_{33} \end{bmatrix} = \begin{bmatrix} 4 & -0.1 & -0.1 \\ -0.1 & 6.8 & 0.3 \\ -0.1 & 0.3 & 1.2 \end{bmatrix}
\]

To identify the mass matrix in the manipulator operation works space according to formulation in (12), Jacobian of the P6 vector (impact point) is yielded as in (37):

\[
Jv_{\alpha} = \begin{bmatrix} C_1(d_4S_2S_6 + d_2) \\ C_1(d_4S_2S_6 + d_2) \\ -d_6C_2S_6C_3 - S_4d_6C_2S_3 \\ C_1(d_2S_6S_2 + S_2C_1) - S_4d_2S_6C_3 \end{bmatrix}
\]
Finally, substituting (12) with evaluated expressions we can identify the required mass matrices in operation space for given robot configurations. The mass matrix numerical representation with obtained eigenvalues is presented in (38):

\[
M_v = \begin{bmatrix}
7.7 & -0.2 & -1.8 \\
-0.2 & 30 & -0.3 \\
-1.8 & -0.3 & 39
\end{bmatrix}
\]

\[
\lambda_1=40, \lambda_2=30, \lambda_3=7.6
\]

\[
\frac{1}{40} = [0 \ 0 \ 1]M_v [0 \ 0 \ 1]
\]

It is seen that the maximum effective mass in the direction of the eigenvector \( V(\lambda_1) = [0,0,1]^T \) (39) (Fig. 8 (a)) can not cause serious injury to a human (sphere H), however, if the configuration/direction is changed, as it is shown in Fig. 8 (b) \( V(\lambda_1) = [1,0,0]^T \), with no variations in the maximum effective mass value, personnel can be under a great risk to be injured.
To estimate the level of this risk we refer to the acceleration and force related danger indices computation assuming that the manipulator interface stiffness $K$ is 250 KN/m and the mass $m$ of a human head is 5 kg.

### 7.1 Acceleration Related Danger Index Computation

From the simulation results it was obtained that the mean duration of an impact $\Delta t$ is 0.025 s. Thus, the critical acceleration value was recomputed with respect to this value in (44):

$$ a_c = \frac{9.81 \times 2 \times \sqrt{\frac{265}{0.025}}}{390} \approx 39 \text{g} $$

Furthermore, basing on acceleration related danger indices evaluated in (26), (32) and considering that the maximum robot effective mass $M$ is 40 kg, boundaries for an initial robot speed were defined:

$$ \frac{M}{m} \sqrt{\frac{M}{m + M}} \leq a_c \; ; \; \nu_0 \leq \frac{390}{\sqrt{\frac{M}{m}}} \approx 0.7 \text{m/s} $$

$$ 2\nu_0 \sqrt{\frac{M}{m + M}} \leq a_c \Delta t \; ; \; \nu_0 \leq \frac{390 \times 0.025}{\sqrt{\frac{M}{2M}}} \approx 0.7 \text{m/s} $$

From the graphical representations below it can be noticed that in spite of the equivalency of two definitions, there is a significant characteristics alteration in the condition when the critical level is overrun. In the HIC based formulation an extent of danger increases much greater. (See Fig. 9 (b))

The impact force that corresponds to the estimated critical acceleration is estimated as 2 KN. This force is more than enough to cause fracture to the human facial bone. In the case of clamping (trapping) the extent of a penetration ($\sigma$) (computed according to (43)), with the facial bone stiffness $K_{fb}=100$ KN/m, can reach 20 mm, that is above the tolerable level.
\[ \sigma = \frac{F}{K_{fb}} \]  \hspace{1cm} (43)

Therefore, this criterion cannot be used under certain critical conditions. To meet more restrictive safety requirements, where no bone whatever stiffness it has can be under the risk to be fractured (or even no pain caused) a force related danger index should be applied.

### 7.2 Force Related Danger Index Computation

Computations were provided on the basis of evaluations presented in (33) for different critical forces including pain tolerance limits and the robot safety standardized requirements. In the (44) and (45) boundaries on the manipulator operating velocities were established based on the pain tolerance (130 N) and maxilla fracture limit forces (660 N) respectively:

\[ v_a \leq 130 \sqrt{\frac{M+m}{M} \frac{1}{M o_x}} = 0.05 m/s \]  \hspace{1cm} (44)

\[ v_s \leq 660 \sqrt{\frac{M+m}{M} \frac{1}{M o_x}} = 0.26 m/s \]  \hspace{1cm} (45)

Figure 10 illustrates the danger index behavioral characteristics according to identified boundaries assuming that the actual manipulator velocity is 0.7 m/s. In this case robot operating safe conditions can be only reached when an arm effective mass is not greater than 3 kg for a pain tolerance (See Fig. 10 (a)) and 14 kg for a maxilla fracture criteria (See Fig. 10 (b)).

With a reference to a safety standard [26], where robot speed should not exceed 0.25 m/s and exerted force -150 N, we can identify that only the force related danger index approach meets requirements of this standard. In the Fig. 11 four indices including standard requirements are represented. The mapping was provided for the robot speeds 0.14, 0.25, 0.6 and 1 m/s. Velocities 0.14 and
0.6 m/s were related to a psychological factor. Experimental researches from different groups [27] showed that at these velocities person does not feel fear or discomfort during interactions with robots. The speed 0.14 m/s was also associated with the first level of interaction (L1, collaborative), while 0.6 m/s was considered for a second level (L2, interior monitoring). From the charts in Fig. 11 it can be seen that at the velocities 1m/s all danger indices exceed the admissible level for the effective mass M=40 kg with interface stiffness $K=200$ KN/m. (See Tab. 2) At 0.6 m/s, only the acceleration related index had positive results. Meanwhile, slow end effector motions were found acceptable for all safety requirements.

Figure 11
Danger indices comparison characteristics: force related (F (130,660 N)), ANSI/RIA standard (F(150 N)) acceleration related ($a_h=39$ g) for chart evaluated for a speed range: 0.14, 0.25, 0.6, 1 m/s

Table 2
Danger Indices Comparative Characteristic

<table>
<thead>
<tr>
<th>Danger Index</th>
<th>$F_c$, N</th>
<th>Me(kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.14m/s</td>
<td>0.25m/s</td>
</tr>
<tr>
<td>$Di(F_{cp})$</td>
<td>130</td>
<td>15</td>
</tr>
<tr>
<td>Standard</td>
<td>150</td>
<td>18</td>
</tr>
<tr>
<td>$Di(F_{cf})$</td>
<td>660</td>
<td>72</td>
</tr>
<tr>
<td>$Di(F_{ah})$</td>
<td>1950</td>
<td>238</td>
</tr>
</tbody>
</table>

"+" indicates the fulfillment of the danger criteria conditions for the manipulator effective mass 40 kg
The results showed that the only force related danger index approach meets requirements stated in the robot safety standard. If it is necessary for a task performance to increase robot speeds, with configuration where robot effective mass is relatively high, then to avoid the risk of serious injury an interface stiffness of this robot has to be lowered. For instance, if we apply a soft rubber material for a robot wrist with the stiffness 100 KN/m, speeds can be raised to the value up to 1 m/s as it is shown in the Fig. 12a (M=40 kg). From the diagram (Fig. 11b) it can be noticed that in this approach a head acceleration is reduced in almost 1,5 times.

![Figure 12](image)

8 Proposed Applications

To keep a “safe” level of interaction on the basis of provided estimations various strategies can be introduced. For instance, for large effective mass configurations, when the danger index is exceeded already at a relatively low velocity, and if there is a possible contact with a human, manipulator trajectory (points) should be redesigned to maintain a tolerable level of danger. Thus, the whole robot path (or, if it is hard to provide, near points) must be hazard free. Fig. 13 (a) illustrates a situation when the manipulator is moving in the direction where the human
presence is not acceptable (danger index circle is violated). In this case, when personnel are detected in the zone, robot control should whether stop the operation or make all possible corrections for the danger reduction. Fig. 13 b illustrates a field where human is allowed to approach a robot operating space. This area is resulted from an intersection of a danger index sphere and an effective mass ellipsoid, represented as a 3-D conic space with an angle $\phi$. For instance, taking danger index associated with a critical pain tolerance force ($F_{cp}=130$ N) where permissive effective mass value is estimated as 8 kg, at a maximum robot operating speed 0.25 m/s along the direction $u$ an angle $\phi$ will be 132 grad (See Fig. 13 b, c). However, zone outside this area should be restricted to prevent any non authorized entering. This approach does not require any on-line changes in the robot configurations or speed during task performance while personnel are inside the “safe” space. Fig. 14 displays a manipulator that is tracking a linear path (from A to B) with low effective mass (inertia) control and constant operating velocity (0.5 m/s here). During this motion a maximum effective mass is changing from 40 to 110 kg, however, effective mass at the direction of the following trajectory $m_u$, is controlled to not exceed the threshold value (8 kg for interaction Level 1). Thereby, this trajectory is “safe” from any harmful impact to a human, even if there is an unexpected robot motion takes place.

Figure 13
Robot safeguarding strategies: a) safety violation b) 3D conic field, c) danger index representation for pain tolerance criteria ($F_{cp}=130$ N)

Figure 14
Robot “safe” path tracking
Conclusion

In the presented work three danger indices were developed and investigated. It was identified that even for very low or minor injury severity levels according to AIS scale, there is a risk to be injured at some critical conditions (trapping). Force related danger index is found to be more appropriate for these situations and closer to the robot safety standard requirements fulfillment. Introduced indices enable to provide analysis on robot operating hazardous characteristics and identify the extent of the potential task/robot related danger. Developed approach allows to human and robot collaborate within all interaction levels maintaining the risk and probability of an accident occurrence at a very low level.

In the future work it is planned to integrate this approach into a safety monitoring system, that would provide the faster and more reliable response of the robot system to the non anticipated failures and hazardous situations.

References


[20] Hertz E.: A Note on the Head Injury Criterion (HIC) as a Predictor of the Risk of Skull Fracture, In 37th Annual Association for the Advancement of Automotive Medicine, Association for the Advancement of Automotive Medicine, 37, Plaines, IL, 1993, pp. 303-312


