The Digital Self-Tuning Control of Step a Down DC-DC Converter

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Abstract: A digital self-tuning control technique of DC-DC Buck converter is considered and thoroughly analyzed in this paper. The development of the small-signal model of the converter, which is the key of the control design presented in this work, is based on the state-space averaged (SSA) technique. Adaptive control has become a widely-used term in DC-DC conversion in recent years. A digital self-tuning Dahlin PID and a Bányász/Keviczky PID controller based on recursive least-squares estimation are developed and designed to be applied to the voltage mode control (VMC) approach operating in a continuous conduction mode (CCM). A comparative study of these two digital self-tuning controllers for step change in input voltage magnitude or output load is also carried out. The simulation results obtained using a Matlab SimPowerSystems toolbox to validate the effectiveness of the proposed strategies are also given and discussed extensively.

Keywords: Continuous Conduction Mode (CCM); Digital Self-tuning Controller; Dahlin PID; Bányász/Keviczky PID; State-Space Averaged (SSA); Voltage Mode Control (VMC)

1 Introduction

Usually power electronic systems consist of one or more power converters which convert one form and/or level of electrical energy into another form or level of electrical energy at the load, thanks to power semiconductor devices controlled by switching action. The advances and availability of modern power semiconductor devices used in power converters have made the switching converter a popular choice in power supplies.

Since the early 1970s, a large number of DC-DC converter circuits have been thoroughly analyzed and designed [1]. Such a converter can increase or decrease the magnitude of the DC voltage and/or invert its polarity. The Buck converter [2],
which uses the switch in series with the supply voltage, is a topology that gives a lower voltage at the load. In contrast, in the topology known as the Boost converter [3], the positions of the switch and inductor are interchanged, which allows this converter to produce an output DC voltage that is greater in magnitude than the input voltage. In the Buck-Boost converter [4], the switch alternately connects the inductor across the power input and output voltages. This converter inverts the polarity of the voltage and can either increase or decrease the voltage magnitude. The Ćuk converter [5] contains inductors in series with the converter input and output ports. The switch network alternately connects a capacitor to the input and output inductors.

The conversion ratio is identical to that of the Buck-Boost converter. Hence, this converter also inverts the voltage polarity, while either increasing or decreasing the voltage magnitude. The single-ended primary inductance converter (SEPIC) can also either increase or decrease the voltage magnitude. However, it does not invert the polarity [6]. These converters are extensively used in electronic equipment such as computer power supplies and battery chargers, and in medical, military, and space applications [1].

The DC-DC converter represents different circuit topologies or configurations within each switching cycle. For the continuous conduction mode (CCM), there are two topologies. For the discontinuous conduction mode (DCM) of operation, a third configuration has to be added to yield a total of three topologies. In each configuration, the system can be described by linear state equations. Switching between the different topologies will vary from cycle to cycle depending on the output of the system, and this further complicates the analysis. This converter presents a nonlinear dynamical due to switching power devices and passive components.

The main approach to modeling DC-DC converters is the state-space averaging method [1], [7], [8]. The averaged continuous-time model uses the duty cycle as an input and describes the system’s slow dynamics, to avoid difficulties posed by the hybrid nature of the system. This model is nonlinear due to the presence of multiplicative terms involving the state variables and the duty cycle. The averaging procedure hides all information about the fast dynamics of the system and fast instabilities (subharmonic oscillations are not captured). In all switching converters, the output voltage is a function of input line voltage, duty cycle, and the load current as well as the converter circuit element values. The scope is to achieve output voltage regulation with voltage mode control (VMC) approach in the presence of input voltage and output load variations.

Many digital controllers using discrete models of converters have been developed in the literature [9], [10] but it is clear that the Proportional-Integral-Derivative (PID) control has been used successfully for regulating processes in industry for more than 60 years. This is so because the design method can be easily grasped and its implementation is very simple.
Such regulators can usually meet demands, but when the dynamic characteristics of the controlled process vary, the PID regulator’s parameters must be readjusted to follow suite. In this case, an adaptive controller should be designed to follow the changes of the operation conditions [11]. The question arises as to how these controllers can be adaptively tuned? In general, self tuners that are capable of automatically adjusting the control loop coefficients are based on recursive least-squares estimation [12], [13].

Many methods and formulae have been developed for tuning the PID controller. Some important examples are the Ziegler and Nichols formula in 1942 [14], the Cohen and Coon formula in 1953 [15], and the Åström and Hägglund in 1985. In 1991 Hang et al. introduced the refined Ziegler and Nichols settings [16], the dominant pole design, proposed by Åström and Hägglund, the internal model control (IMC) design method [17], the ITAE integral of the time weighted absolute error [18], and ISE integral of the squared error. There was also the optimal formulae and the gain and phase margin (GPM) design method [19]. The Dahlin controller was proposed by Dahlin [20] and Higham [21], independently. The Dahlin controller is a distinctive algorithm for the control of single input/single output (SISO) plants with dead time. With advances in digital hardware and digital control techniques, it is becoming feasible to implement control schemes, such as self-tuning control for power converters. In recent years there has been increasing interest in the development of efficient control strategies to improve the dynamic behavior of systems by using digital self-tuning controllers. The Bányász/Keviczky digital self-tuning PID controller was proposed by Cs. Bányász and L. Keviczky in 1982. This algorithm is based on the explicit identification of a second order process model with time delay [22], [23].

![Synoptic scheme of the VMC for the Buck converter](image)

The Dahlin digital self-tuning PID is also used in this study. The major advantage of this algorithm is the reduction of the tuning from three to two parameters [24].
Hence, the contribution of this paper is to introduce a mathematical model using the state-space averaged (SSA) technique for the Buck converter in continuous conduction mode (CCM).

A comparative study between two digital self-tuning PID controllers, the Dahlin PID and the Bányász/Keviczky PID, is carried out.

In this paper, VMC approach for Buck converter has been analyzed and developed using these two controllers. The simulation results using a MATLAB SimPowerSytems toolbox to validate the effectiveness of the proposed control strategies are also included.

2 A State-Space Averaged Model of Buck Converter

The basic power circuit of Buck topology is shown in Figure 1. It consists of two semiconductor devices: a controlled power device, such as a power MOSFET or IGBT and an uncontrolled device, such as a power diode and passive elements. They consist basically of an inductor in series with a parallel combination of a capacitor and resistor [25].

The state space model is given by the following equation:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

While taking as state vector $x = [i_L \ v_o]^T$

In the case where the switch $S$ is closed and the diode $D$ is opened, state space model matrices $A_1$, $B_1$, $C_1$ during $d$ of the switching time became:

$$A_1 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{v_o}{L} \\ 0 \end{bmatrix}, C_1 = [0 \ 1]$$

In the second case where the switch $S$ is opened and the diode $D$ is closed, state space model matrices $A_2$, $B_2$, $C_2$ exist during $1-d$ of the switching time interval:

$$A_2 = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, C_2 = [0 \ 1]$$
Combining both set of matrix by using the equation (1) we obtain:

\[
\dot{x} = \left[ A_1 d + A_2 (1 - d) \right] x(t) + \left[ B_1 d + B_2 (1 - d) \right] u(t)
\]

The nonlinear equation (4) leads to:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
\frac{R_l}{L} & -\frac{1}{L} & x_1 \\
\frac{1}{C} & -\frac{1}{RC} & x_2 \\
\end{bmatrix} + \begin{bmatrix}
d \\
L \\
0 \\
\end{bmatrix} v_i
\]

The SSA technique introduces a small AC modulating signal represented by "\(^\wedge\)" in the DC steady-state quantities (which are represented by the upper case letters) [26].

Therefore:

\[
d = D + \hat{d}
\]

\[
x = X + \hat{x}
\]

\[
v_i = V_i + \hat{v}_i
\]

Inserting equations (6) through (8) in equations (4) and recognizing that in steady state, \( \dot{X} = 0 \). Moreover in the AC equation, terms containing products of \( \dot{x} \) and \( \hat{d} \) can be neglected (small multiplied by small gives an even smaller result). Based on these facts, the DC and AC equations can be obtained as follows:

**DC equation:**

\[
0 = A_0 x + B_0 v_i
\]

**AC equation:**

\[
\dot{x} = A_0 \hat{x} + B_0 \hat{v}_i + E \hat{d}
\]

Where:

\[
A_0 = \left[ A_1 D + A_2 (1 - D) \right]
\]

\[
B_0 = \left[ B_1 D + B_2 (1 - D) \right]
\]

\[
E = \left[ (A_1 - A_2) X + (B_1 - B_2) V_i \right]
\]

Back to the notation of equations (6) through (8), we finally have:

\[
\dot{x} = A_0 x + B_0 v_i + E \hat{d}
\]

Combining the matrix according to equation (14) gives:
From equation (15), we obtain a set of two linearized equations corresponding to the small-signal model:

\[
\begin{align*}
\dot{x}_1 &= -\frac{R_L}{L} x_1 - \frac{1}{L} \dot{x}_2 + \frac{D}{L} \dot{v}_i + \frac{\dot{d}}{L} V_i \\
\dot{x}_2 &= \frac{1}{C} \dot{x}_1 - \frac{1}{RC} \dot{x}_2
\end{align*}
\]

(16)

(17)

\(D\) represents the static duty cycle given by \(D = \frac{V_{\text{out}}}{V_{\text{in}}}\).

The transfer function relating the output voltage to the duty cycle is given by:

\[
\frac{\tilde{x}_i(s)}{d(s)} = \frac{V_i}{LCS^2 + \left(\frac{C}{R} + \frac{L}{R}\right) s + 1 + \frac{R_L}{R}}
\]

(18)

The transfer function of the system with a zero-order hold is given by:

\[
\begin{align*}
\tilde{x}_2(z) &= B(z^{-1})A(z^{-1})^{-1} = \frac{b_0 z^{-1} + b_1 z^{-2} + b_2 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2}} \\

\tilde{x}_2(z) &= Z\left\{1 - e^{sT_i} \tilde{x}_2(s)\right\} = (1 - z^{-1}) Z\left\{1 - e^{sT_i} \tilde{x}_2(s)\right\} \\
\end{align*}
\]

(19)

(20)

Where:

\[
\begin{align*}
b_0 &= \frac{V_i}{LC} \left[A_1 + A_2 + A_3\right] = 0 \\
b_1 &= \frac{V_i}{LC} \left[-(A_1 + A_3)e^{sT_i} - (A_1 + A_2)e^{sT_i} - A_2 - A_3\right] \\
b_2 &= \frac{V_i}{LC} \left[A_1 e^{s(T_1 + T_2)} + A_2 e^{sT_i} + A_3 e^{sT_i}\right] \\
a_1 &= -e^{sT_i} - e^{sT_i} \\
a_2 &= e^{s(T_1 + T_2)}
\end{align*}
\]

(21)

And:
\[
\begin{align*}
A_1 &= \frac{1}{s_1 s_2} \\
A_2 &= \frac{1}{s_1 (s_1 - s_2)} \\
A_3 &= \frac{1}{s_2 (s_2 - s_1)}
\end{align*}
\]

(22)

\(s_1, s_2\) represent the poles of the system.
\(T_s\) represents the sampling time.

### 3 Voltage Mode Control

In the DC-DC converter, the output supply must be adjusted to be constantly equal to a fixed value; considering that the input supply and load can vary, this control is called VMC (Voltage Mode Control). In this way the link between the input and the output of the system can be modified by a controller addition, which is generally calculated according to certain criteria defining the type of desired response for the system in closed loop (stability, response time, …).

Figure 1 shows the synoptic scheme of the VMC for the Buck converter, the control signal \(v_c\) fed into a pulse-width modulator (PWM) that compares \(v_c\) with the signal \(v_i\) (saw wave). The modulator produces a switched voltage waveform that controls the gate of the semiconductor \(S\). The duty cycle \(d\) of this waveform is proportional to the control voltage \(v_c\).

### 4 Digital Self-Tuning Controller

A digital self-tuning controller is a controller that during each sample interval performs three major steps shown in Figure 2.
- Estimates the parameters of the discrete plant model.
- Calculates the controller parameters using the estimated plant model parameters.
- Calculates and implements the new control signal.

In this work, two digital self-tuning controllers based on recursive least-squares estimation are used to improve the robustness of the controlled system.
4.1 Digital Self-Tuning Bányász/Keviczky PID Controller

Discrete PID regulators can be implemented in many different ways. Different structures correspond to different continuous PID regulators. The most common sampled data PID regulator is given by the discrete transfer function:

\[
C(z) = \frac{U(z)}{E(z)} = \frac{Q(z^{-1})}{P(z-1)}
\]  

(23)

\[
Q(z^{-1}) = q_0 + q_1 z^{-1} + q_2 z^{-2}
\]  

(24)

\[
P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2}
\]  

(25)

Where \(q_0, q_1, q_2, p_1\) and \(p_2\) are the controller parameters.

The controlled process is described by the discrete transfer function:

\[
G_p(z) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 (1 + \gamma z^{-1})}{1 + a_1 z^{-1} + a_2 z^{-2} z^{-d}}
\]  

(26)

With:

\[
\gamma = \frac{b_2}{b_1}, \text{ where } b_1 \neq 0 \text{ and } d > 0 \text{ is the discrete time delay of the process.}
\]

Since the process is stable and a second order model is used, a very good robust design idea is to choose \(Q(z^{-1})\) proportional to the denominator of the estimated process model:
Which means that
\[ q_1 = q_0a_1, \quad q_2 = q_0a_2 \]  \hspace{1cm} (28)
Which in turn means all poles are canceled and that it is applicable for stable processes only. In practical tuning, this means that the regulator cancels the two largest time constants in the process dynamics [26], and this idea allows us to simplify the control loop of the system:
\[ C(z^{-1})G_p(z^{-1}) = \frac{k_I(1+\gamma z^{-1})}{1-z^{-1}}z^{-d} \]  \hspace{1cm} (29)
Where the integrator gain is
\[ k_I = q_0b_1 \]  \hspace{1cm} (30)
The coefficients of the controller are given by:
\[ \gamma = \frac{b_2}{b_1} \]
\[ q_0 = \frac{k_I}{b_1} \]
\[ q_1 = q_0a_1 \]
\[ q_2 = q_0a_2 \]
\[ p_1 = -1+\gamma \]
\[ p_2 = -\gamma \]
For \[ \gamma = 0 \rightarrow k_I = \frac{1}{2d-1} \]  \hspace{1cm} (32)
For \[ \gamma > 0 \rightarrow k_I = \frac{1}{2d(1+\gamma)(1-\gamma)} \]  \hspace{1cm} (33)
if \[ \gamma < 1 \], equation (32) is used together with a serially connected digital filter, which is easy to be realized in digital control systems [27]:
\[ G_p(z) = \frac{1}{1+\gamma z^{-1}} \]  \hspace{1cm} (34)
From Equation (23) the control law can be written as follows:
\[ U(z) = \frac{Q(z^{-1})}{P(z^{-1})}E(z) \]  \hspace{1cm} (35)
By inserting polynomials (24) and (25) into equation (35), the relations that calculate the controller output become:

\[ u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) - p_1 u(k-1) - p_2 u(k-2) \]  

(36)

### 4.2 Digital Self-Tuning Dahlin PID Controller

The velocity PID control algorithm is represented by the equation:

\[
\begin{align*}
&u(k) = u(k-1) + K_p \left[ e(k) - e(k-1) + \frac{T_s}{T_i} e(k) + \frac{T_d}{T_s} (e(k) - 2e(k-1) + e(k-2)) \right] \\
&\quad + K_p \left[ e(k) - e(k-1) + \frac{T_s}{T_i} e(k) + \frac{T_d}{T_s} (e(k) - 2e(k-1) + e(k-2)) \right] \\
&\quad + K_p \left[ e(k) - e(k-1) + \frac{T_s}{T_i} e(k) + \frac{T_d}{T_s} (e(k) - 2e(k-1) + e(k-2)) \right]
\end{align*}
\]

(37)

In the adaptive control scheme, the controller tuning parameters \( K_p \), \( T_i \), and \( T_d \) are computed directly from the parameters of the second-order model of the process by the controller synthesis method proposed by Dahlin [20], [11]. The adapter formulas developed by this method for a process modeled by equation (19) require that this controller uses parameters estimation vector in the form:

\[ \Theta^F(k) = [a_1, a_2, b_1] \]  

(38)

And the parameter \( b_2 \) must be forced to zero by the estimator. Assuming this to be the case, the following relationships have been developed for the tuning parameters of the PID algorithm:

\[
\begin{align*}
K_p &= \frac{(a_1 + 2a_2)Q}{b_1} \\
T_i &= -\frac{T_s}{\left(a_1 + 2a_2\right) + 1 + \frac{T_d}{T_s}} \\
T_d &= \frac{T_s a_2 Q}{K_p b_1}
\end{align*}
\]

(39)

For a PI algorithm, \( a_2 \) must also be forced to zero so that \( T_d = 0 \).

The variable \( Q \) is defined by:

\[ Q = 1 - e^{-\tau_i/B} \]  

(40)

Where \( B \) is the tuning factor that represents the time constant of the desired closed-loop response. The smaller value of \( B \) leads to a faster response of the closed control loop.

The relation that calculated the controller output is given by equation (36).
To identify the unknown coefficients of the controller, equation (36) is equal to equation (37) as follows:

\[
q_0 = K_p \left( 1 + \frac{T_x}{T_s} + \frac{T_d}{T_s} \right)
\]

\[
q_1 = -K_p \left( 1 + 2 \frac{T_d}{T_s} \right)
\]

\[
q_2 = K_p \frac{T_d}{T_s}
\]

\[
p_1 = -1, \quad p_2 = 0
\]

(41)

5 Simulation Results

In order to compare the performance of these two digital PID self-tuning regulators using a VMC control methodology of the Buck converter, some simulation results are given with the system parameters:

\[ V_i = 15 \text{volts}, \quad L = 3.716mH, \quad R_v = 0516\Omega, \quad C = 100\mu F, \quad R = 7.5\Omega. \] Switching frequency \( f = 20\text{KHz} \).

The parameters of the digital self-tuning Bányász/Keviczky PID Controller are:

Dead time \( d = 5 \), \( T_s = 10^{-5} \).

The parameters of the design of the digital self-tuning Dahlin PID are:

Adjustment factor \( B = 9.10^{-4} \), \( T_s = 10^{-6} \).

Figure 3 shows the output voltage response obtained by the digital self-tuning controllers Bányász/Keviczky PID and Dahlin PID. It is clear that the output voltage response obtained via the Bányász/Keviczky PID is faster than that obtained by the Dahlin PID. It is found that spikes occur in the output voltage controlled by the Bányász/Keviczky PID more than in that obtained with the Dahlin PID.
To verify the effectiveness of the proposed controllers, first the system responses are driven initially with 15\textit{volts} as the input voltage when a step change in the input voltage from 15\textit{volts} to 23\textit{volts} is applied at $t = 0.1s$ as can be seen from Fig. 4.

![Figure 4](image-url)

**Figure 4**
Output voltage responses under step input voltage change from 15\textit{volts} to 23\textit{volts} at $t = 0.1s$
Figure 5 shows the zoom of the output voltage response of Figure 4 obtained by the digital self-tuning controllers Bányász/Keviczky PID and Dahlin PID. It is clear that the voltage increase is recovered more quickly with the self-tuning digital Dahlin PID controller for this sudden change in the input voltage than with the Bányász/Keviczky PID.

To check the hardiness of the digital self-tuning Dahlin PID controller, a very important change in the input voltage from 15 volts to 30 volts is applied at \( t = 0.1 \text{s} \) as can be seen from Figure 6. It is clear that the output voltage response obtained by the Dahlin PID is faster than that obtained by the Bányász/Keviczky PID.
Finally, we examine a crucial aspect of the controllers’ operation, namely the system’s behaviour against low load current. The load resistor is increased from its nominal value $7.5\,\Omega$ to $10\,\Omega$ at $t = 0.1\,s$. The simulation results in Figure 7 show that the voltage drop is recovered more quickly with the self-tuning digital Dahlin PID controller for this sudden change in the load than with the Bányaész/Keviczky PID.

![Figure 7](image1.png)

**Figure 7**
Output voltage response under step load change from $7.5\,\Omega$ to $10\,\Omega$ at $t = 0.05\,s$

![Figure 8](image2.png)

**Figure 8**
Zoom of the output voltage response of Figure 7

**Conclusion**

In this paper the state-space averaged technique to derive the small-signal model of the DC-DC Buck converter is made.

Two digital self-tuning Bányaész/Keviczky PID and Dahlin PID controllers based on recursive least-squares estimation are designed to regulate the output voltage using a VMC strategy.
Comparative studies were made with these two digital self-tuning PID controllers for a sudden change in input voltage magnitude and/or load change. The digital self-tuning Dahlin PID controller gives the better performance and is more robust for model inaccuracies and disturbances in comparison with the Bányaśz/Keviczky PID controller.

Simulated results obtained with a MATLAB SimPowerSytems toolbox validate the effectiveness of the proposed control strategy.

References


