Improved Delay-range-dependent Stability Analysis of a T-S Fuzzy System with Time Varying Delay

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Abstract—This paper presents a new & improved stability analysis for Takagi-Sugeno (T-S) Fuzzy system subjected to interval time-varying delay. The stability analysis provides a sufficient stability condition in an Linear Matrix Inequality (LMI) frame work that can estimate less conservative delay upper bound for a given lower bound of delay. The analysis is carried out by proposing new and modified Lyapunov-Krasovskii (L-K) functional along with the judicious use of Jensen’s integral for eliminating the integral terms arising out from the time derivative of L-K functional, these modifications in turn leads to convex combination of the LMIs. Few numerical examples are included to illustrate the effectiveness of the obtained stability condition compared to some recently published stability conditions for different delay nature while delay is varying in intervals.

I. INTRODUCTION

One usually assumes that the future state of the dynamical system is determined solely by the present state of the system and is independent of the past state information, whereas the class of system that includes past state information along with present state for finding the future state is referred as time-delay system (TDS) [2], [1], [7]. The ubiquitous presence of time-delay in any physical system (e.g., chemical processes, biological processes, process control, population dynamics and aerospace engineering etc) is known to be a source of instability and performance degradation of the system [2], [5], [7]. Assessment of stability in TDS involves computing the delay bound up to which system can retain stability, in sequel there are two ways for stability assessment for such systems - (i) time-domain technique and (ii) frequency-domain technique. Former technique has relative merit of ease in controller synthesis and computational ease although it yields conservative delay bound result, but the later method can compute exact delay bound but due to computational complexity controller synthesis is difficult. The time-domain technique is adopted in this work for stability assessment as there is still room for increasing the delay bound. The method is based on Lyapunov’s second method referred as L-K functional approach which subsequently formulates the stability condition in an LMI frame work. For physical system it is natural to expect that system will lose stability at a certain finite delay value, thus the derived stability condition contains the information of delay size in it - such conditions are referred in the literature as delay-dependent stability condition. Hence, vast research literature on stability analysis for TDS is directed towards deriving delay-dependent one because of its physical significance and can be found in [3], [5] and references therein. Continuous improvement in the delay-dependent stability results are reported where attempts are made to reduce the conservativeness in the estimate of delay upper bound compared to the existing methods can be found in [5] and references there in. Recently in [6], [9], [10], [32] another notion of stability for such system has been coined called delay-range dependent stability. According to this notion, the delay lower bound is not restricted to zero but it is a small positive number thus giving the measure of delay range (difference between upper & lower delay value) due to its practical significance like in Networked control system [9], [7].

Delay-range-dependent stability conditions are derived using L-K functional approach by two popular techniques - (i) by partitioning the delay range, which is referred here as delay-partitioning (DP) or also referred as delay decomposition approach [39] and (ii) by not partitioning the delay range, that is referred here as non delay-partitioning (NDP) method [40], [41], [45], [42], [43], [11], [44]. Literature reveals that partitioning a delay interval (DP approach) or adopting augmented LK functional involves more free matrix, whereas in NDP method as no sub division of delay interval is carried out so it involves lesser number of free matrix variables compared to the former case. The relative merit of the DP over NDP approach is that former method yields less conservative delay range compared to the later method due the involvement of lesser decision (matrix) variables.

Stability of time-delayed system on linear systems is, by itself, a challenging issue being an infinite dimensional system, and Takagi-Sugeno (T-S) fuzzy model have been extensively used for this purpose [17], [18], [19], [20], [21], [22], mainly due to their approximation capabilities and the ability to extend several results from linear systems to nonlinear ones [23]. Several approaches developed for linear time-delay system have been borrowed to deal with fuzzy time delay system as revealed from the literature, but it is worth to mention here that the number of subsystem dealt in this case are more
so it becomes a difficult job to search for common feasible positive symmetric definite and free matrices. This requires to undertake certain modification over the analysis developed for general time-delay systems. In recent years, the problems of stability and stabilization of the T-S fuzzy systems with time delay have attracted rapidly growing interest [24], [39], [30], [29], [41]. More specifically, related to the problem of checking stability for T-S fuzzy systems with interval time-varying delays one can find recent works as [37], [38] and references therein.

In this paper, new stability analysis for T-S systems subject to interval time-varying delay are proposed. These conditions make use of an appropriately chosen L-K functional, judicious use of integral inequalities for uncertain and certain limits of integration and convex combination properties in order to reduce the conservativeness. In addition, the proposed methodology allows us to check the stability of state-delayed T-S fuzzy system with interval time-varying delay whose delay variation is either slow or fast.

II. PROBLEM STATEMENT

Consider a fuzzy T-S system with time-varying delay with the \( i \)th rule of the system of the form represented below [37], Plant rule \( i \):

\[
\text{IF } z_i(t) \text{ is } M_{i1} \text{ and } \ldots \text{ z}_p(t) \text{ is } M_{ip} \text{ THEN } \\
\dot{x}(t) = A_i x(t) + A_{di} x(t - d(t)) \\
x(t) = \phi_i(t), \ t \in [-d, 0], \ i = 1, 2, \ldots r \quad (1)
\]

where, \( z_1(t), z_2(t), \ldots, z_p(t) \) are the premises variables, \( M_{ij} \) with \( i = 1, 2, \ldots r \), \( j = 1, 2, \ldots p \) are the fuzzy sets, \( x(t) \in \mathbb{R}^n \) is the state vector, \( \phi_i(t) \) is the vector valued initial function, \( A_i \) and \( A_{di} \) are the constant real matrices of appropriate dimensions. A positive scaler \( r \) depicts the number of IF-THEN fuzzy rule considered, while \( d(t) \) is the time-varying delay. The fuzzy time-delay system in (1) satisfies following conditions,

\[
0 < d_1 < d(t) \leq d_2, \ \forall \ t \geq 0 \quad (2)
\]

\[
\dot{d}(t) \leq \mu < \infty \quad (3)
\]

The nature of the delay is categorized as slow and fast varying delays based on \( \mu < 1 \) and \( \mu \geq 1 \) respectively, both the natures of delays are treated here. In (2), \( d_1 \) and \( d_2 \) are the lower and upper bound of the delay.

Given \( x(t) \) the final output of the fuzzy system is inferred as follows,

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \{ A_i x(t) + A_{di} x(t - d(t)) \} \\
= \sum_{i=1}^{r} w_i(z(t)) \{ A_i x(t) + A_{di} x(t - d(t)) \} \quad (4)
\]

where, \( z(t) = [z_1(t) \ z_2(t) \ldots z_p(t)]^T \), \( h_i(t) = \Pi_{j=1}^{p} M_{ij}(z_j(t)) \) and \( w_i(z(t)) = \sum_{z \in \Xi} w_i(z) \forall t \). The term \( M_{ij}(z(t)) \) is the grade of membership of \( z_j(t) \) in \( M_{ij} \).

Following assumptions are standard for fuzzy T-S model [37] and references therein regarding the membership function \( w_i(z(t)) \),

\[
h_i(z(t)) \geq 0, \sum_{i=1}^{r} h_i(z(t)) \geq 0 \\
w_i(z(t)) \geq 0, \sum_{i=1}^{r} w_i(z(t)) = 1, i = 1, 2, \ldots, r \quad (5)
\]

Hereafter for simplicity we write the membership function \( w_i(z(t)) = w_i \). In view of the above, system represented in
(4) can be rewritten as,
\[ \dot{x}(t) = \tilde{A}x(t) + \tilde{A}_d x(t - d(t)), \]
\[ x(t) = \sum_{i=1}^{r} w_i \phi_i(t), \quad t \in [-d, 0] \tag{6} \]
where,
\[ \tilde{A} = \sum_{i} w_i A_i, \text{ and } \tilde{A}_d = \sum_{i} w_i A_{di} \]
In this work we will develop sufficient stability condition in an LMI framework for T-S fuzzy time-delay system (6) with delay varying in intervals with an emphasis on fast varying delay nature.

III. STABILITY ANALYSIS

In this section we derive new and improved delay-range-dependent stability criteria without partitioning the delay range (NDP approach) unlike in [37]. Following modification has been incorporated in obtaining the stability condition and is referred as judicious use of integral inequalities in the present work.

**Remark 1:** The bounding of the integral terms arising out of L-K functional derivative will be approximated using free matrix approach [26] as:

\[ -\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta) R \dot{x}(\theta) d\theta \leq \left[ \begin{array}{c} x(t-\alpha) \\ x(t-\beta) \end{array} \right]^T \begin{bmatrix} M + M^T & -M + N^T \\ -M - N^T & \end{bmatrix} \begin{bmatrix} M \\ N \end{bmatrix} R^{-1} \begin{bmatrix} M \\ N \end{bmatrix}^T \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}, \tag{7} \]
where ‘∗’ represents symmetric components, \( R = R^T > 0, \beta > \alpha \geq 0, \gamma = \beta - \alpha > 0 \) and \( M, N \) are free weighting matrices of appropriate dimension. However, in [9], it has been shown that use of such free weighting matrices may impose constraint on the resulting stability criterion and obtain less conservative results by using an integral type inequality (Jensens’s inequality) of [2] given by,

\[ -\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta) R \dot{x}(\theta) d\theta \leq \gamma^{-1} \left[ \begin{array}{c} x(t-\alpha) \\ x(t-\beta) \end{array} \right]^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}. \tag{8} \]

Many attempts have been made to deduce equivalency and conservativeness of several criteria based on either (7) or (8), e.g. see [4], [9]. Explicit relation between (7) and (8) can be established following the equivalency results in [13]. In this regard, note that, the first term in the right-hand-side (RHS) of (7) may be represented as:

\[ \begin{bmatrix} M \\ N \end{bmatrix} \begin{bmatrix} I \\ -I \end{bmatrix}^T + \begin{bmatrix} I \\ -I \end{bmatrix} \begin{bmatrix} M \\ N \end{bmatrix}^T. \tag{9} \]

Now, it is easy to see that (7) and (8) are equivalent in view of Theorem 4.1 of [13]. Moreover, the RHS of (7) is minimum when

\[ M = M^T = -N = -N^T = -\gamma^{-1} R, \tag{10} \]
and for such a choice (7) becomes (8).

From the above, it seems that use of (8) is always desired since it does not involve additional free variables besides being equivalent to (7). However, if \( \gamma \) is uncertain and required to be approximated with its lower or upper bound then use of (7) would be beneficial since the choice (10) can not be met with an approximated \( \gamma \). Moreover, the RHS of (7) is affine on the parameter \( \gamma \), which is beneficial in formulating convex combinations of LMIs.

**Theorem 1:** Given a fuzzy time-delay system (6) satisfying the conditions (2)-3 and considering the assumption (5) the system is asymptotically stable, if there exist real symmetric positive-definite matrices \( P, Q_k, k = 1, 2, ..., 4 \), \( R_m > 0, m = 1, 2 \) and any free matrices \( \Phi_{li}, l = 1, 2, \ldots, r \) of appropriate dimensions with scalars \( d_1, d_2 \) and \( \bar{d} = (d_2 - d_1) \) representing delay lower bound, delay upper bound and delay range respectively, such that the following LMIs are satisfied,

\[ \Theta_i + \Phi_{li} R_2^{-1} \Phi_{li}^T < 0, \quad i = 1, 2, ..., r, \quad l = 1, 2 \tag{11} \]

where,

\[ \Theta_i = \begin{bmatrix} \Theta_{11i} & \Theta_{13i} & 0 \\ \Theta_{22i} & \Theta_{33i} & 0 \\ \Theta_{44i} & \end{bmatrix} \]

\[ \Theta_{11i} = PA_i + A_i^T P + \sum_{k=1}^{3} Q_k - R_1 + A_i^T (\bar{d}_2 R_1 + R_2) A_i, \]
\[ \Theta_{13i} = PA_{di} + A_i^T (\bar{d}_2 R_1 + R_2) A_{di}, \]
\[ \Theta_{22i} = Q_4 - Q_1 - R_1 + \bar{d}_1 T_{i1} + T_{i2}^T, \]
\[ \Theta_{33i} = -(1 - \mu) (Q_5 + Q_4) + \bar{d}_1 (N_{1i} + N^T_{1i} - T_{2i} - T_{2i}^T), \]
\[ + A_i^T (\bar{d}_2 R_1 + R_2) A_{di}, \quad \Theta_{34i} = \bar{d}_1 (N_{1i} + N^T_{1i}), \]
\[ \Theta_{44i} = -Q_2 + \bar{d}_2 (N_{2i} + N^T_{2i}). \]

**Proof** Define a L-K functional candidate for the system (6) as,

\[ V(t) = x^T(t) P x(t) + V_2 + V_3 + V_4 + V_5 + V_6 \tag{12} \]

where,

\[ V_2 = \sum_{k=1}^{2} \int_{t-d_k}^{t} x^T(\theta) Q_k x(\theta) d\theta \]
\[ V_3 = \int_{t-d_1}^{t-d_2} x^T(\theta) Q_3 x(\theta) d\theta \]
\[ V_4 = \int_{t-d_1}^{t-d_2} x^T(\theta) Q_4 x(\theta) d\theta \]
\[ V_5 = d_1 \int_{t-d_1}^{t-d_2} \int_{0}^{\bar{d}} x^T(s) R_1 x(s) ds d\theta \]
\[ V_6 = \bar{d}^{-1} \int_{t-d_1}^{t-d_2} \int_{0}^{\bar{d}} x^T(s) R_2 x(s) ds d\theta \]
Time-derivative of (12) along the state trajectory of (6) is,
\[
\dot{V}(t) = 2x^T(t)P\dot{x}(t) + 2x^T(t)P\dot{A}x(t - d(t))
+ \sum_{k=1}^{3} x^T(t)Q_kx(t) - x^T(t - d(t))(Q_1)x(t - d(t))
+ x^T(t - d1)(Q_4)x(t - d1)
- \sum_{k=3}^{4} (1-d(t))x^T(t - d(t))Q_kx(t - d(t))
- x^T(t - d2)Q_2x(t - d2)
+ \dot{x}^T(t)(d_1^2R_1 + R_2)\dot{x}(t)
- d_1 \int_{t-d_1}^{t} \dot{x}^T(\theta)R_1\dot{x}(\theta)d\theta
- d_1^{-1} \int_{t-d_2}^{t-d_1} \dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta
\]
Next, for the last integral term of (13), the following identity
\[
\begin{aligned}
&= \int_{t-d_1}^{t} \dot{x}^T(\theta)R_1\dot{x}(\theta)d\theta \\
&\leq \left[ \begin{array}{c}
\dot{x}(t) \\
\dot{x}(t - d_1)
\end{array} \right]^T
\left[ \begin{array}{ccc}
-R_1 & R_1 & x(t) \\
* & -R_1 & x(t - d_1)
\end{array} \right]
\end{aligned}
\]
where
\[
\rho = \frac{d(t) - d_1}{d_1}, \quad 0 \leq \rho \leq 1,
\]
\[
\zeta_1 = \left[ \begin{array}{c}
(x(t - d_1))^T \\
(x(t - d(t)))^T
\end{array} \right]^T
\]
and
\[
\zeta_2 = \left[ \begin{array}{c}
(x(t - d(t))^T \\
x(t - d_2))^T
\end{array} \right]^T.
\]
Substituting the integral terms of (13) by (14) and (15), and then using (16) and (17) one obtains,
\[
\dot{V}(t) \leq \xi^T(t)(\Theta_i + \rho\Phi_1R_2^{-1}\Phi_1^T + (1-\rho)\Phi_2R_2^{-1}\Phi_2^T)\xi(t),
\]
where
\[
\xi(t) = \left[ \begin{array}{c}
x(t)^T \\
x(t - d_1)^T \\
x(t - d(t))^T \\
x(t - d_2)^T
\end{array} \right]^T.
\]
Then the stability requirement of (6) becomes
\[
\Theta_i + \rho\Phi_1R_2^{-1}\Phi_1^T + (1-\rho)\Phi_2R_2^{-1}\Phi_2^T < 0,
\]
this can be rewritten as
\[
\rho(\Theta_i + \Phi_1R_2^{-1}\Phi_1^T) + (1-\rho)(\Theta_i + \Phi_2R_2^{-1}\Phi_2^T) < 0
\]
Clearly, in view of (19), the left hand side (LHS) of (22) is a polytope of matrices and is always negative definite if its two certain vertices are so. Hence, (22) can be equivalently written as:
\[
\Theta_i + \Phi_1R_2^{-1}\Phi_1^T < 0 \quad (23)
\]
\[
\Theta_i + \Phi_2R_2^{-1}\Phi_2^T < 0 \quad (24)
\]
Finally using Schur Complement on (23) one can obtain (11), this completes the proof.

Remark 2: If $d_1 = 0$, then $d = d_2 - 0 = d_2$ thus Theorem 1 leads to following corollary,

Corollary 1: Given a fuzzy time-delay system (6) satisfying the conditions (2)-(3) and considering the assumption (5) the system is asymptotically stable, if there exist real symmetric positive-definite matrices $P$, $Q_k$, $k = 2,3, R_2 > 0$, and any free matrices $\Phi_i$, $l = 1,2$, $i = 1,2..r$ of appropriate dimensions such that the following LMIs are satisfied,
\[
\hat{\Theta}_i + \Psi_iR_2^{-1}\Psi_i^T < 0, \quad i = 1,2..r, \quad l = 1,2
\]
where
\[
\hat{\Theta}_i = \left[ \begin{array}{ccc}
\hat{\Theta}_{11i} & \hat{\Theta}_{12i} & 0 \\
* & \hat{\Theta}_{22i} & \hat{\Theta}_{23i}
\end{array} \right]
\]
and
\[
\begin{aligned}
\hat{\Theta}_{11i} &= PA_i + A_i^TP + \sum_{k=2}^{3} Q_k + A_i^TR_2A_i \\
+ d_2^{-1}(T_{11i} + T_{21i}) \\
\hat{\Theta}_{12i} &= PA_{di} + A_{di}^TPA_{di}d_2^{-1}(-T_{11i} + T_{21i}) \\
\hat{\Theta}_{22i} &= -(1-\mu)Q_{33} + A_{di}^TR_2A_{di}d_2^{-1}(-N_{11i} + N_{21i}) \\
\hat{\Theta}_{23i} &= d_2^{-1}(-N_{11i} + N_{21i} - T_{21i}) \\
\hat{\Theta}_{33i} &= -Q_2 - d_2^{-1}(N_{21i} + N_{22i})
\end{aligned}
\]
\[
\Psi_i = \left[ \begin{array}{ccc}
T_{11i} & T_{21i} & 0 \\
0 & N_{11i} & N_{21i}
\end{array} \right]^T.
\]
TABLE I
MAXIMUM DELAY UPPER BOUND (d2) OF EXAMPLE 1 FOR μ < 1

<table>
<thead>
<tr>
<th>Methods</th>
<th>d1 = 0.1</th>
<th>d1 = 0.5</th>
<th>d1 = 0.9</th>
<th>d1 = 1</th>
<th>d1 = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[37]</td>
<td>1.3451</td>
<td>1.4598</td>
<td>1.5903</td>
<td>1.6557</td>
<td>2.1182</td>
</tr>
<tr>
<td>Theorem 1</td>
<td>1.7241</td>
<td>1.8286</td>
<td>1.9808</td>
<td>1.9974</td>
<td>2.3751</td>
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TABLE II
MAXIMUM DELAY UPPER BOUND (d2) OF EXAMPLE 1 FOR μ ≥ 1

<table>
<thead>
<tr>
<th>Methods</th>
<th>d1 = 0.2</th>
<th>d1 = 0.4</th>
<th>d1 = 0.6</th>
<th>d1 = 0.8</th>
<th>d1 = 1</th>
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<tbody>
<tr>
<td>[27]</td>
<td>0.687</td>
<td>0.850</td>
<td>0.946</td>
<td>1.048</td>
<td>NR</td>
</tr>
<tr>
<td>[28]</td>
<td>0.7945</td>
<td>0.8487</td>
<td>0.9316</td>
<td>1.0325</td>
<td>NR</td>
</tr>
<tr>
<td>[29]</td>
<td>1.141</td>
<td>1.150</td>
<td>1.172</td>
<td>1.209</td>
<td>NR</td>
</tr>
<tr>
<td>[30]</td>
<td>0.9119</td>
<td>0.9773</td>
<td>1.0639</td>
<td>1.1662</td>
<td>NR</td>
</tr>
<tr>
<td>[47]</td>
<td>1.1639</td>
<td>1.1734</td>
<td>1.1994</td>
<td>1.2552</td>
<td>NR</td>
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<tr>
<td>Theorem 1</td>
<td>1.4865</td>
<td>1.5065</td>
<td>1.5579</td>
<td>1.6370</td>
<td>1.7324</td>
</tr>
</tbody>
</table>

IV. ILLUSTRATIVE EXAMPLES

The effectiveness of the proposed stability condition can be judged by the delay upper bound results obtained for a given lower bound compared to the recent existing results using delay partitioning approach.

Example 1: Consider a time-delayed fuzzy system in [37].

Plant Rule 1:
If x(t) is M1 then,
\[
\dot{x}(t) = A_1 x(t) + A_{d1} x(t - d(t))
\]

Plant Rule 2:
If x(t) is M2 then,
\[
\dot{x}(t) = A_2 x(t) + A_{d2} x(t - d(t))
\]

The membership function for plant rule 1 and 2 are,
\[
M_1(x(t)) = \left\{ \begin{array}{l}
1 - \frac{1}{1 + e^{-5(x_1(t) - (\pi/6))}} \\
1 + e^{-5(x_1(t) - (\pi/6))}
\end{array} \right.
\]
\[
M_2(x(t)) = 1 - M_1(x(t))
\]

where, \(x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}^T\),
\[
A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}
\]
and \(A_2 = \begin{bmatrix} -1.5 & 0 \\ 0 & -0.75 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}^{-0.85} \).

For the system considered in Example 1 using corollary 1 we could obtain \(d_2 = 1.4393\) with \(d_1 = 0\) for \(\mu \geq 1\), whereas most recent result reported in [38] could establish \(d_2 = 0.8833\). The other most recent existing results in [8], [30], [46] for this system under identical parameter values yields \(d_2 = 1.2696, 0.9899, 1.278\) respectively irrespective of the approach adopted. The delay upper bound results for a given \(d_1\) (delay lower bound) for fast varying delay compared to existing results are presented in Table I and II respectively.

Example 2: Consider a time-delayed fuzzy system in [38],

Plant Rule 1:
If x(t) is M1 then,
\[
\dot{x}(t) = A_1 x(t) + A_{d1} x(t - d(t))
\]

Proof: Setting \(Q_1 = Q_4 = R_1 = 0\) in L-K function (12) and proceeding in the similar fashion as in Theorem 1 leads to new augmented state vector \(\xi(t) = [x^T(t) \ x^T(t - d(t)) \ x(t - d_2)]^T\) of reduced dimension, which ultimately yields the LMIs in (25). As the derivation is straightforward following Theorem 1, so it is omitted for the sake of brevity.

V. CONCLUSION

In this paper a new & improved delay-range-dependent sufficient stability condition for a fuzzy time-delay system has been proposed without partitioning the delay into subintervals. More the number of subsystems, the existence of common symmetric positive definite matrices and free matrices becomes difficult. In view of this fact, modification in the selection of free matrices has been incorporated so as to reduce the conservativeness of the delay bound results.

The efficacy of the stability analysis is ascertained by the delay upper bound \(d_2\) results presented in Table I - III compared to some recent existing methods, as results of the proposed method is even superior than that of delay-partitioning approach.
Robust stability and stabilization for such system will be dealt by the authors as a future work following the same analytical framework.

REFERENCES