Experimental Results of Evolving Takagi–Sugeno Fuzzy Models for a Nonlinear Benchmark

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Abstract—The paper offers evolving Takagi–Sugeno (T–S) fuzzy models for a nonlinear benchmark represented by the pendulum-crane system and focused on the dynamics of pendulum angular position behavior. The rule bases and parameters of T–S fuzzy models are continuously evolved by an online identification algorithm in terms of computing the potentials of new data points. Accepting the pendulum angle as model output, two T–S fuzzy models are derived, viz. with a single input and with three inputs. The experimental results are obtained for a pendulum-crane laboratory equipment system and they show the performance of the proposed evolving T–S fuzzy models.

I. INTRODUCTION

Evolving Takagi–Sugeno (T–S) fuzzy systems are derived by learning on the basis of online clustering for rule base learning, and the model structure and parameters can evolve similarly [1]–[3]. Several online clustering and recursive parameter estimation algorithms are recently reported in the literature such as semi-supervised classification algorithms [4], incremental fuzzy rule-based systems [5], drift and shift handling in on-line data streams [6], robust incremental FLEXFIS approaches [7], recursive Gustafson-Kessel clustering and fuzzy least squares algorithms [8], complexity reduction of evolving T–S fuzzy systems [9], evolving classifiers combined with tree-based user profiling [10], and sliding mode control integrated in incremental learning algorithms [11].

This paper presents two evolving T–S fuzzy models with one and three inputs for a nonlinear benchmark represented by the pendulum-crane system. The fuzzy models are derived by the cost-effective implementation and application of the online identification algorithm suggested in [12]. The identification algorithm is supported by continuously evolving rules and parameters in terms of computing the potentials of new data points. The results given in this paper contribute to the better understanding of how fuzzy systems can effectively adapt and coevolve with nonlinear systems in the framework of Cognitive Infocommunications (CogInfoCom) [13], [14]. These aspects are also treated in combination with cognition processes in [15] and with robotics systems in [16]. The nonlinear system identification by evolving T–S fuzzy models is also addressed in [17], and nonlinear processes are discussed in [15], [18]–[27].

Experimental results validate the performance of the proposed evolving T–S fuzzy models by the application of the algorithm on a pendulum-crane laboratory equipment system. The dynamics of the pendulum angular position is targeted and used as model output.

This paper is structured as follows. Section II shows the proposed approach to obtain the evolving T–S fuzzy models. Section III carries out the experimental validation of the new fuzzy models. Section IV points out the concluding remarks.

II. APPROACH TO OBTAIN TAKAGI–SUGENO FUZZY MODELS

The state equations of the process in pendulum-cart systems, as a representative nonlinear benchmark with the structure proposed and built according to [28], are

$$
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= \left\{ \frac{J_p}{(m_c + m_p) l_{mm}} \left[ \frac{p-m}{(m_c + m_p) l_{d}} - x_4^2 \sin x_2 \right] - \left( \frac{f_x - p_c}{m_c + m_p} \right) \frac{x_1}{l_{d}} \right\} \cos x_2 + \left\{ \frac{f_x x_4}{(m_c + m_p) l_{d}} \right\} \sin x_2, \\
\dot{x}_4 &= \left\{ \frac{p-m}{(m_c + m_p) l_{d}} - x_4^2 \sin x_2 \right\} \cos x_2 + \left\{ \frac{f_x x_4}{(m_c + m_p) l_{d}} \right\} \sin x_2, \\
\end{align*}
$$

where the variables and parameters are: $x_1$ – the cart position, $x_2$ – the angle between the upward vertical and the ray pointing at the center of mass cart, $x_3$ – the cart velocity, $x_4$ – the pendulum angular velocity, $u$ – the control signal represented by a constrained PWM voltage signal, $|u| \leq u_{\text{max}} > 0$, $m_c$ – the equivalent mass of the
cart, $m_p$ – the mass of the pole and load, and $l_p$ – the distance from the axis of rotation to the center of mass. Introducing the cart velocity as the third state variable $x_3$ and the pendulum angular velocity as the fourth state variable $x_4$, $J_p$ – the moment of inertia of the pendulum-cart system with respect to the axis of rotation, $p_i$ – the ratio between the control force and the control signal, $p_3$ – the ratio between the control force and $x_3$, $f_p$ – the rotational friction coefficient, and $f_c$ – the dynamic cart coefficient. The first state variable $x_1$ is the distance between the cart and the center of the rail. The parameter values in the experimental setup are [28], [29]

\[
u_{\text{max}} = 0.5, \quad m_c = 0.76 \text{ kg}, \quad m_p = 0.052 \text{ kg}, \quad l_d = 0.011 \text{ m}, \quad J_p = 0.00292 \text{ kg\cdot m}^2, \quad p_i = 9.4 \text{ N},
\]

$$p_3 = -0.548 \text{ N\cdot s/m}, \quad f_p = 6.65 \cdot 10^{-5} \text{ N\cdot m\cdot s/ rad},$$

$$f_c = 0.5 \text{ N\cdot s/m}.$$

The expression of the rule base of T–S fuzzy models for this nonlinear benchmark is

Rule $i$: IF($z_i$ IS $L_{i1}$) AND ... AND ($z_n$ IS $L_{in}$) THEN ($y_i = a_{i0} + a_{i1}z_1 + \ldots + a_{in}z_n$), $i = 1...nR$,

where: $nR$ – the number of rules, $z_i$, $j = 1...n$ – the input variables, $n$ – the number of input variables, $z = [z_1, z_2, \ldots, z_n]^T$ – the input vector, $T$ – the notation for matrix transposition, $L_{ij}$, $i = 1...nR$, $j = 1...n$ – the input linguistic terms, $y_i$ – the output of the local crisp system model $y_i = a_{i0} + a_{i1}z_1 + \ldots + a_{in}z_n$ in the rule consequent of rule $i$, $i = 1...nR$, and $a_{ii}, i = 1...nR$, $l = 0...n$ – the parameters in the rule consequents. The fuzzy sets of the input linguistic terms $L_{ij}$, $i = 1...nR$, $j = 1...n$ are modeled by Gaussian membership functions with the membership degrees

\[
\mu_{ij}(z_i) = \exp[-(4/r_s^2)(z_i - z_{ij})^2], \quad i = 1...nR, \quad j = 1...n, \quad (4)
\]

where: $r_s$, $r_c > 0$ – the spread of all input membership functions and $z_{ij}$, $i = 1...nR$, $j = 1...n$ – the centers of input membership functions $\mu_{ij}$, $i = 1...nR$, $j = 1...n$.

For the algebraic product used as the AND operator, each rule generates the firing degree $\tau_i(z)$

\[
\tau_i(z) = \text{AND}(\mu_{i1}(z_1), \mu_{i2}(z_2), \ldots, \mu_{in}(z_n))
\]

\[
= \mu_{i0}(z_1) \cdot \mu_{i1}(z_2) \cdot \ldots \cdot \mu_{in}(z_n).
\]

The application of the weighted average defuzzification method results in the fuzzy model output

\[
y = \frac{\sum_{i=1}^{nR} \sum_{j=1}^{n} \tau_i(z_j) y_i}{\sum_{i=1}^{nR} \sum_{j=1}^{n} \tau_i(z_j)} = \sum_{i=1}^{nR} \lambda_i y_i = \sum_{i=1}^{nR} \lambda_i [1 \ z_i^T]^T \pi_i
\]

where: $\lambda_i = \tau_i / \sum_{i=1}^{nR} \tau_i$, $i = 1...nR$ – the normalized firing degree of rule $i$, and $\pi_i = [a_{i0}, a_{i1}, a_{i2}, \ldots, a_{in}]^T$, $i = 1...nR$ – the parameter vector of rule $i$.

A data point is modeled by vector $p$ in the input-output data space $R^{n+1}$

\[
p = [z \ y]^T = [z_1, z_2, \ldots, z_n, y]^T
\]

\[
= [p^1, p^2, \ldots, p^n, p_{n+1}]^T \in R^{n+1}.
\]

The online identification algorithm uses the potential of data point $p_i$ at time step $k$ defined in [12] as the first order Cauchy type function

\[
P_k(p_k) = 1/1 + \frac{1}{k-1} \sum_{l=1}^{k-1} (d_{lk})^2
\]

\[
d_{lk} = p_l - p_k, \quad k = 2...D,
\]

where: $d_{lk}$ – the projection of the distance between $p_l$ and $p_k$, on the axis $p^j$, $j = 1,\ldots,n+1$, and $D$ – the number of data points. The recurrent equation which gives the new data point potential is [12]

\[
\theta_k = \sum_{j=1}^{n+1} (p_j)^{2}, \quad \sigma_k = \sum_{j=1}^{n+1} (p_j)^{2}, \quad \nu_k = \sum_{j=1}^{n+1} (p_j)^{2}.
\]

A least squares approach is employed in the computation of the parameters in the rule consequents, and $y$ in (6) is expressed as follows in this approach:

\[
y = \psi^T \theta, \quad \psi^T
\]

\[
= [\lambda_1 [1 \ z_1^T]^T, \lambda_2 [1 \ z_2^T]^T, \ldots, \lambda_n [1 \ z_n^T]^T]^T,
\]

\[
\theta = [\pi_1^T, \pi_2^T, \ldots, \pi_n^T]^T.
\]

For the set of input-output data $\{p_k | k = 1\ldots D\}$, a global objective function in this approach is

\[
J_G = \sum_{k=1}^{D} \left(y_k - \psi^T \theta \right)^2, \quad \psi^T = [\lambda_1 [1 \ z_1^T]^T, \lambda_2 [1 \ z_2^T]^T, \ldots, \lambda_n [1 \ z_n^T]^T]^T
\]

\[
\lambda_i(z_k) [1 \ z_k^T]^T, \quad \lambda_n(z_k) [1 \ z_n^T]^T
\]

The minimization of $J_G$ can be achieved by a recursive least squares algorithm with the recurrent equations and initial conditions [30], [31]

\[
\hat{\theta}_k = \hat{\theta}_{k-1} + C_k \psi_k^T (y_k - \psi_k^T \hat{\theta}_{k-1}),
\]

\[
C_k = C_{k-1} - \frac{C_k \psi_k \psi_k^T C_{k-1}}{1 + \psi_k^T C_{k-1} \psi_k}, \quad k = 2\ldots D,
\]

\[
\hat{\theta}_k = [\pi^T_1, \pi^T_2, \ldots, \pi^T_n]^T = [0 \ 0 \ \ldots \ 0]^T,
\]

\[
C_k = \Omega \ I,
\]

where: $C_k \in R^{n(n+1)\times n(n+1)}$ – the covariance matrix, $\Omega$ – the $nR(n+1)^{th}$ order identity matrix, $\Omega = \text{const}$, $\Omega > 0$ – a large number, and $\hat{\theta}_k$ – an estimation of the parameters in the rule consequents at time step $k$.
The approach to obtain evolving T–S fuzzy models is supported by the online identification which consists of the following steps formulated from [12] taking into account the previous notations.

**Step 1.** The rule base is initialized with a single rule, \( n_R = 1 \). The subtractive clustering approach [30] can be applied in order to obtain the fuzzy model parameters using the first data point, \( \mathbf{p}_1 \). The parameters are initialized by (12) and by

\[
k = 1, \ n_R = 1, \ \mathbf{z}_1^* = \mathbf{z}_1^\ast, \quad P_1(\mathbf{p}_1) = 1.
\]  

(13)

where: \( \mathbf{z}_1^* \) – the center of cluster 1 (i.e., first cluster center), and \( \mathbf{z}_1^\ast \) – the center of rule 1. \( \mathbf{z}_1^* \) also represents a projection of the data point \( \mathbf{p}_1^* \) on the axis \( \mathbf{z} \).

**Step 2.** \( k \) is set to \( k = k + 1 \) at next step time. The next data point \( \mathbf{p}_k \) is read.

**Step 3.** The potentials of all new data points are calculated recursively in terms of (9).

**Step 4.** The potentials of the centers of existing the rules are updated recursively using [32]

\[
P_k(\mathbf{p}_1^*) = [(k - 1)P_{k-1}(\mathbf{p}_1^*)]/(k - 2 + P_{k-1}(\mathbf{p}_1^*))
\]

\[+ P_{k-1}(\mathbf{p}_1^*) \sum_{j=1}^{n_R} (d_{kj}^*)^2],
\]

(14)

where: \( P_l(\mathbf{p}_1^*) \) – the of the cluster center. This cluster is also a prototype of rule \( l \).

**Step 5.** The possible modification of the rule base structure is performed using the potential of the new data points. This potential is compared to the potential of all existing rule centers. The rule base structure is modified if the condition pointed out in the sub-step 5.1 is fulfilled.

**Sub-step 5.1.** If the potential of the new data point is larger than that of existing centers, i.e.:

\[
P_k(\mathbf{p}_k) > P_k(\mathbf{p}_l^*), \quad i = 1...n_R.
\]

(15)

and this point is close enough to an old cluster center, i.e.:

\[
P_k(\mathbf{p}_k) \geq \max_{l=1}^{n_R} P_k(\mathbf{p}_l^*) - \delta_{\text{max}} / r_c > 1,
\]

(16)

\[
\delta_{\text{max}} = \min_{i=1}^{n_R} \| \mathbf{z}_i - \mathbf{z}_i^* \|^2, \quad i = 1...n_R.
\]

(17)

where: \( \delta_{\text{max}} \) – the distance between this new data point and the closest of all existing rule centers, supposed to have the index \( l \), the parameter \( r_c = \text{const} \), \( r_c > 0 \), determines the radius of the vicinity which will have measurable potential alleviations because of the small distance to an existing rule center, \( r_c = r_c / 1.5 \) as suggested in [31], then the new data point \( \mathbf{p}_k \) replace the old one. This new data point plays the role of a prototype of the rule center, supposed to have the index \( j \):

**Step 6.** The rule consequent parameters are updated recursively updated using (12).

**Step 7.** The evolving T–S fuzzy model output at next time step \( k + 1 \) is predicted by the particular form of (10)

\[
\hat{y}_{k+1} = \mathbf{w}_j^\ast \hat{\mathbf{h}}_k.
\]

(22)

The algorithm continues with step 2 until all data points are read. Step 1 is carried out offline, and steps 2 to 7 are carried out online.

The dynamics is introduced in the T–S fuzzy models of the pendulum-crane system by considering different delayed values of the control signal \( u \) and eventually of the pendulum angular position (pendulum angle) \( y = \dot{x}_5 \). Two such versions of delayed values are considered, and this leads to two fuzzy models with the output \( \hat{y}_k \) for both fuzzy models, the input \( u_{k-1} \) for the T–S fuzzy model 1 and the inputs \( u_{k-1}, u_{k-5}, y_{k-5} \) for the T–S fuzzy model 2. Other combinations of inputs are possible for many nonlinear processes [34]–[39].
III. EXPERIMENTAL VALIDATION OF TAKAGI–SUGENO FUZZY MODELS

The online identification algorithm presented in Section II has been applied to obtain the two evolving T–S fuzzy models. Setting the sampling period $T_s = 0.01\,s$, the control signal $u$ has been generated according to Fig. 1 in order to cover different ranges of magnitudes.

This input of the nonlinear benchmark has been applied to the laboratory equipment which in order to generate the input-output data points $(z_k, y_k), k = 1, \ldots, D$. Fig. 1 gives a total number of 600 data points which is separated in training data and validation data. The first $D = 300$ data points (the time frame from 0 s to 30 s) which result from Fig. 1 are validation data, and the rest of $D = 300$ data points (the time frame from 30 s to 60 s) which result from Fig. 1 are testing data.

The seven steps of the identification algorithm have been applied to derive the six evolving T–S fuzzy models. The value of the parameter $\Omega$ at step 1 has been set to $\Omega = 10000$.

The first T–S fuzzy model has evolved to $nR = 2$ rules. Table I gives the parameter values of this fuzzy model, obtained after the application of the online identification algorithm for $n = 1$.

The responses of pendulum position $y$ versus time of the first evolving T–S fuzzy model and of the nonlinear benchmark (viz., experimental response of the laboratory equipment) are presented in Fig. 2 which illustrates the results for both training and validation data. The sum of squared errors between the pendulum position of the first T–S fuzzy model and of the nonlinear benchmark for the validation data is 100.4562.

<table>
<thead>
<tr>
<th>Rule number $i$</th>
<th>$z_i^*$</th>
<th>$r_i$</th>
<th>$a_{i0}$</th>
<th>$a_{i1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−0.2</td>
<td>0.4</td>
<td>−6.1455</td>
<td>1.2291</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.4</td>
<td>−5.9461</td>
<td>−1.1892</td>
</tr>
</tbody>
</table>

The responses of pendulum position versus time of the first Takagi–Sugeno fuzzy model and for nonlinear benchmark.
The second T–S fuzzy model has evolved to $nR = 2$ rules. The parameter values of this fuzzy model, obtained after the application of the online identification algorithm for $n = 3$, are presented in Table II.

The responses of pendulum position $y$ versus time of the second evolving T–S fuzzy model and of the nonlinear benchmark are presented in Fig. 3 for training and validation data. The sum of squared errors between the pendulum position of the second T–S fuzzy model and of the nonlinear benchmark for the validation data is 73.7885.

<table>
<thead>
<tr>
<th>Rule number $i$</th>
<th>$x_{i1}$</th>
<th>$x_{i2}$</th>
<th>$x_{i3}$</th>
<th>$r_i$</th>
<th>$a_{i0}$</th>
<th>$a_{i1}$</th>
<th>$a_{i2}$</th>
<th>$a_{i3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.4</td>
<td>-0.1378</td>
<td>-0.0275</td>
<td>-0.0275</td>
<td>0.8997</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.4</td>
<td>-0.1468</td>
<td>-0.0293</td>
<td>-0.0293</td>
<td>1.0077</td>
</tr>
</tbody>
</table>

The results presented in Fig. 2 and Fig. 3 and the value of the sum of squared errors used as performance index illustrate the performance enhancement of the second T–S fuzzy model. The performance of the evolving fuzzy systems can be further improved by considering several combinations of delayed system inputs and outputs [40].

IV. CONCLUSION

This paper has proposed two evolving T–S fuzzy models dedicated to a representative nonlinear benchmark which deals with crane systems. The models are obtained by the application of an online identification algorithm which gives continuously evolving rule bases and parameters targeting the pendulum angle dynamics.

The performance and experimental validation of the evolving T–S fuzzy models for the given nonlinear benchmark are advantageous as they can be used in the process modeling and development of model-based fuzzy controllers in various applications. Such applications include control systems and structures in CogInfoCom and in other fields [41]–[46].

Future research will target several stable and optimal design fuzzy control system designs for this benchmark and for other nonlinear processes. The data-driven experiment-based calculation of objective function gradients and the stability conditions viewed as constraints will be considered in the design methodologies.

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