Abstract - In the paper after a short review of basic type-2 terms and some related possibilities in T2 approximate reasoning process are given. The introduced Mamdani type of fuzzy approximate reasoning uses the fuzziness of fuzzy sets for the weighting of observed rule. To the describing reasoning process based on type-2 fuzzy sets the 3D representation of those sets is given.

I INTRODUCTION

In fuzzy control systems the typical fuzzy approach is as follows: for measured fuzzy input data, and given rule base system an output signal should be generated to reach the better state of the controlled system using fuzzy approximate reasoning method. If the fuzzy control mechanism uses type-2 fuzzy sets or type-2 fuzzy logic and inference, it is called a type-2 (T2) fuzzy system.

Since the introducing of the fuzziness of fuzzy values [3], there are periods of different intense activity in type-2 fuzzy set theory and applications investigation. In recent years we can find in [1],[2] summary of basic terms of T2 fuzzy systems, and there are several published results related to the reasoning methods based on T2 fuzzy systems [4],[5].

Type-2 fuzzy sets and their properties help as to solve a problem for many applications, where type-1 fuzziness of uncertain linguistic terms are used and have a non-measurable domain of FS-s, or results imprecise boundaries. The question is coming up: how should be the fuzziness of fuzzy sets and fuzzy operators effected on the fuzzy approximate reasoning process?

In the paper after a short introduction about type-2 FS and basics of Mamdani type fuzzy approximate reasoning, a possible influence of the fuzziness of fuzzy sets involved in approximate reasoning model is given. The basic idea is, that in the reasoning process the observed rule in the rule base system is weighted by the measure of fuzziness at the highest point of the common area of the rule premise and system input.

II TYPE-2 FUZZY SETS

Type-2 and higher-types fuzzy sets (FS) eliminate the paradox of type-1 fuzzy systems which can be formulated as the problem that the membership grades are themselves precise real numbers. Type-1 fuzzy set (T1 FS) has grade of membership that is crisp, whereas a type-2 fuzzy set (T2 FS) has grade of membership that are fuzzy, so T2 FS are „fuzzy-fuzzy” sets.

The 2-D representation of fuzzy membership of fuzzy sets is the footprint representation of uncertainty (FOU), which is given with the uncertainty about left and right end point of the left side of the membership function, and with the uncertainty about left and right end point of the right side of the membership function.

Let be \( x \in X \) from the universe of basic variable of interest. Let be \( MF(x) \) the T1 fuzzy membership function of the fuzzy linguistic variable or other fuzzy proposition. Functions \( UMF(x) \) and \( LMF(x) \) are functions of the left-end and right-end point uncertainty. In a fix point \( x' \) of the universe \( X \) it is possible define so called vertical slices of the uncertainty, describing it for different possibilities of the \( MF(x) \) functions \((i=1,2,...,N)\), included in the shading of FOU.

In this case, for example if we have Gaussian primary membership function (MF), very often the uniform shading over the entire FOU means the uniform weighting, possibilities. T2 FS with FOU representation and uniform possibilities on FOU is called interval type-2 FS (IT2 FS).

The second way is to use 3D representation, where in the domain \( xOy \) the F1 FS \( A(x) \) is represented, and in the third dimension for every crisp membership value \( A(x) \) of the basic variable \( x \) a value of possibility (or uncertainty) is given as the function \( MF(x,A(x)) = \mu(x,A(x)) \). It is the embedded 3D T2 FS (Fig.1.). On the Fig.1. the value \( \mu(x,A(x)) \) is a Gaussian distribution value to represent the uncertainty of fuzz set. This uncertainty is the lowest at the kernel of the T1 FS.
III  MAMDANI TYPE APPROXIMATE REASONING

In Mamdani-based FLC the model the rule output \( B'_i(y) \) of the \( i^{th} \) rule if \( x \) is \( A_i \), then \( y \) is \( B_i \) in the rule system of \( n \) rules is represented usually with the expression

\[
B'_i(y) = \sup_{x \in X} \{ T(A'_i(x), T(A_i(x), B_i(y)) \}
\]

where \( A'_i(x) \) is the system input, \( x \) is from the universe \( X \) of the inputs, and \( y \) is from the universe of the output. For a continuous associative t-norm \( T \), it is possible to represent the rule consequence model by

\[
B'_i(y) = T \left( \sup_{x \in X} T(A_i(x), A'_i(x)), B_i(y) \right)
\]

The consequence (rule output) is given with a fuzzy set \( B'(y) \), which is derived from rule consequence \( B_i(y) \), as an upper bounded, cutted membership function derivated from the of the \( B_i(y) \). The cut,

\[
DOF_i = \sup_{x \in X} T(A_i(x), A'_i(x))
\]

is the generalized degree of firing level of the rule, considering actual rule base input \( A'_i(x) \), and usually depends on the covering over \( A_i(x) \) and \( A'_i(x) \), i.e. on the \( \sup \) of the membership function of \( T(A'_i(x), A_i(x)) \).

Rule base output, \( B'_{out} \) is an aggregation of all rule consequences \( B'_i(y) \) from the rule base. As aggregation operator usually an S conorm fuzzy operator is used.

\[
B'_{out}(y) = S(B'_{i1}(y), S(B'_{i2}(y), S(\ldots, S(B'_{in}(y), B'_i(y))))).
\]

The crisp FLC output \( y_{out} \) is constructed as a crisp value calculated with a defuzzification method.

Approximate reasoning based on distance based operators

Because of the properties of the distance-based operators [6], it was unreasonable to use the classical degree of firing, to give expression to coincidence of the rule premise (fuzzy set \( A_i \)), and system input (fuzzy set \( A'_i \)), therefore a degree of coincidence (Doc) for these fuzzy sets has been initiated. It is nothing else, but the proportion of area under membership function of the modified entropy-based intersection of these fuzzy sets, and the area under membership function of their union (using \( \text{max} \) as the fuzzy union).

The rule output fuzzy set \( B'_i \) should achieved as a cut of rule consequence \( B_i \) with Doc.

\[
B'_i(y) = \text{max}(B_i(y), \text{Doc}_i)
\]

where \( \text{Doc}_i \) is the degree of coincidence, and gives expression to coincidence of the rule premise (fuzzy set \( A_i \)), and system input (fuzzy set \( A'_i \)) in the \( i^{th} \) rule of the rule system:

\[
\text{Doc}_i = \frac{\int T_{e}^{\text{min}}(A_i(x), A'_i(x))dx}{\int \text{max}(A_i(x), A'_i(x))dx}.
\]

The FLC rule base output is constructed as crisp value calculated from associative using t-conorm on all rule outputs \( B'_i(y) \) [7].

IV APPROXIMATE REASONING IN T2 ENVIRONMENT

Let assume, that we have T2 fuzzy sets as the rule premise of the rules and as the system input, described respectively by

\[
MFA_i(x, A_i(x)) = \mu(x, A_i(x))
\]

and

\[
MFA'_i(x, A'_i(x)) = \mu(x, A'_i(x))
\]

where the fuzziness represent the uncertainness of T1 fuzzy membership values of these sets.

Let take into the consideration the fuzziness of these fuzzy sets. Calculating the rule output it is possible to introduce weightiness at the \( i^{th} \) rule proportionally with the fuzziness of rule premise and rule input in the considered rule. The gain value \( G_i \) (weightiness of the observed \( i^{th} \) rule) can be calculated as the maximum fuzziness at \( x \in X \), where

\[
DOF_i = \sup_{x \in X} T(A_i(x), A'_i(x)) = T(A_i(x), A'_i(x))
\]

regarding \( MFA_i(x, A_i(x)) \) and \( MFA'_i(x, A'_i(x)) \), i.e.

\[
G_i = \max(MFA_i(x, A_i(x)), MFA'_i(x, A'_i(x)))
\]

The weighted \( i^{th} \) rule output is calculated by the

\[
B'_i(y) = G_i \cdot T(DOF_i, B'_i(y)).
\]

If we apply the distance based operators in the reasoning process, the gain \( G_i \) can be the maximum of the fuzziness at the level \( e \), where \( e \) is the unit element of the operator.

Example

Applying Matlab Fuzzy toolbox, and using Gaussian distribution for the representation of the fuzziness of the rule premise and system input, based on the basic operators of min and max, the control surfaces are presented:

- In Fig. 3 without weighted rule outputs,
- In Fig. 4 with weighted rule outputs.

The investigated system is a MISO system, shown on the Fig. 2.
V Conclusion

After a short introduction about type-2 FS and basics of Mamdani type fuzzy approximate reasoning, a possible influence of the fuzziness of fuzzy sets involved in approximate reasoning model was given. In the reasoning process the observed rule in the rule base system is weighted by the measure of fuzziness at the highest point of the common area of the rule premise and system input. The control surfaces show that the gained reasoning process returns more powerful control than the classical type of reasoning process.

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References


