Abstract—In this paper we propose non-parametric t-norms such as algebraic, trigonometric and Hamacher product, furthermore parametric Hamacher t-norm in Mamdani type inference systems. Various models with trapezoidal shaped fuzzy membership function are applied in order to improve the efficiency of bacterial memetic algorithm in automatic fuzzy rule identification.

I. INTRODUCTION

In the design of fuzzy controllers is inevitable the calculus of fuzzy rules [11]. It is well known that the extraction of important fuzzy rules from a given rule base depends crucially from the application of various global and local search algorithms. The application of the Bacterial Memetic Algorithm (BMA) for fuzzy rule base identification (FRBI) was proposed in [1, 2]. In our previous paper [6] we studied the following bacterial type algorithms: the Bacterial Evolutionary Algorithm (BEA) [3], and the Modified Bacterial Memetic Algorithm (MBMA) [4]. The bacterial memetic algorithms involve the Levenberg-Marquardt (LM) method [9, 10], a local search algorithm, with high convergence speed.

In [2] the standard “min” fuzzy t-norm as aggregation operator was proposed, and for defuzzification the center of gravity (COG) method [1] was applied.

Examples with numerical simulations can be found in [6]. The function approximation capabilities of the identified rule base systems from the required rule base size, furthermore from the systems convergence speed point of view were compared. The data sets were derived from one and multidimensional test functions, furthermore from data samples of real world applications.

The efficiency of the various bacterial type algorithms were tested by Mamdani type inference system with trapezoidal shaped fuzzy membership functions in case of selected non-parametric t-norms.

The goal is to develop an efficient, reliable and rapid method for automatic fuzzy rule identification, able to reduce computational effort and processing time. In this paper we propose various non-parametric t-norms and parametric Hamacher t-norm instead of “min” fuzzy operator in the Mamdani type inference system.

The Hamacher operator, and its special case, the algebraic operator has been studied because of their advantageous properties [12]. Furthermore a new trigonometric t-norm [5] which consists of simple combinations of trigonometric functions has been used.

The paper is organized as follows: after the Introduction, in Section II the fuzzy rule base identification process is presented. We propose various t-norms as aggregation operators in FRBI. The derivatives for the Jacobian computation, essential in the Levenberg-Marquardt method, are determined. Finally, some concluding remarks are given.

II. FUZZY RULE BASE EXTRACTION USING SELECTED T-NORMS

In case of Mamdani type inference system the relative importance of the $j^{th}$ fuzzy variable in the $i^{th}$ rule is [6]

$$\mu_{ij}(x_i) = \frac{x_i - a_{ij}}{b_{ij} - a_{ij}} N_{ij,1}(x_i) + \frac{d_{ij} - x_i}{d_{ij} - c_{ij}} N_{ij,2}(x_i),$$

(1)

where $a_{ij}$, $b_{ij}$, $c_{ij}$ and $d_{ij}$ are the breakpoints of the trapezoidal shaped fuzzy membership functions, $a_{ij} \leq b_{ij} \leq c_{ij} \leq d_{ij}$ must hold, and

$$N_{ij,1}(x_i) = \left[ \begin{array}{ll} 1, & \text{if } x_i \in [a_{ij}, b_{ij}], \\ 0, & \text{otherwise} \end{array} \right], \quad N_{ij,2}(x_i) = \left[ \begin{array}{ll} 1, & \text{if } x_i \in [c_{ij}, d_{ij}], \\ 0, & \text{otherwise} \end{array} \right].$$

(2)

The activation degree of the $j^{th}$ rule (if the t-norm is the minimum):

$$w_i = \min_{j=1}^{n} \mu_{ij}(x_i)$$

(3)

where $n$ is the number of input dimensions. Then the COG method is applied as the defuzzification method.

In the next several ways of modeling are suggested by substituting in Mamdani type inference system the “min” t-norm by various non-parametric and parametric t-norms as aggregation operators. We propose the algebraic and trigonometric norms, furthermore the Hamacher operator. As a second step, the derivatives of the multiple argument aggregation expressions (part of the FRBI process) are determined.

A. Algebraic t-norm

The algebraic t-norm is:
The activation degree of the $i$th rule (aggregation operator) with three input variables using algebraic t-norm will be [6]:

$$w_i = \mu_{1i} \cdot \mu_{2i} \cdot \mu_{3i}$$  

In general:

$$w_i = \prod_{j=1}^{i} \mu_{ji} = P_i$$  

The derivatives expression of $w_i$ used in Jacobian computation are [7]:

$$\frac{\partial w_i}{\partial \mu_{ji}} \cdot \frac{P_i}{\mu_{ji}} = \frac{1}{\mu_{ji}} \cdot w_i$$ if $w_i > 0$.

B. Trigonometric t-norm

The trigonometric norms [5] in combination with the standard negation were applied for defining fuzzy flip-flop neurons, the basic units in fuzzy flip-flop based neural networks. The new trigonometric t-norm is defined as:

$$i_f(a, b) = \frac{2}{\pi} \arcsin \left( \sin \left( a \frac{\pi}{2} \right) \sin \left( b \frac{\pi}{2} \right) \right)$$

In general:

$$w_i = \frac{2}{\pi} \arcsin \left( \prod_{j=1}^{i} \sin \left( \mu_{ji} \frac{\pi}{2} \right) \right)$$

Where: $P_i = \prod_{j=1}^{i} \sin \left( \mu_{ji} \frac{\pi}{2} \right)$

The derivatives of $w_i$ will be:

$$\frac{\cos \left( \mu_{ji} \frac{\pi}{2} \right)}{\sin \left( \mu_{ji} \frac{\pi}{2} \right)} \cdot \frac{P_i}{\sqrt{1 - P_i^2}}$$

if $0 < w_i < 1$.

C. Hamacher t-norm

The expression of the Hamacher t-norm is [8]:

$$i_h(a, b) = \frac{a \cdot b}{v + (1 - v)(a + b - a \cdot b)}$$

where $\nu \in (0, \infty)$

1. If $\nu = 1$, the Hamacher t-norm expression is $i_H(a,b) = a \cdot b$ which corresponds to the algebraic t-norm ($i_A$), see subsection A.

2. If $\nu = 0$, the Hamacher t-norm is called Hamacher product ($i_{HP}$) which corresponds to the Dombi t-norm [8], when the Dombi parameter $a = 1$.

For three input variables the aggregation operator expression [6] is:

$$w_i = \frac{2}{\pi} \arcsin \left( \sin \left( \mu_{1i} \frac{\pi}{2} \right) \sin \left( \mu_{2i} \frac{\pi}{2} \right) \sin \left( \mu_{3i} \frac{\pi}{2} \right) \right)$$

When $\nu = 0$, the aggregation operator with three input variables is
The activation degree of the \(i\)th rule in case of four variables:

\[
w_i = \frac{\mu_{j_1} \cdot \mu_{j_2} \cdot \mu_{j_3}}{\mu_{j_1} \cdot \mu_{j_2} + \mu_{j_1} \cdot \mu_{j_3} + \mu_{j_2} \cdot \mu_{j_3} - 2\mu_{j_1} \cdot \mu_{j_2} \cdot \mu_{j_3}}
\]

(15)

The activation degree of the \(i\)th rule in case of four variables:

\[
w_i = \frac{A}{B + C + D + E - 3 \cdot A}
\]

where

\[
B = \mu_{j_1} \cdot \mu_{j_2} \cdot \mu_{j_3}
\]
\[
C = \mu_{j_1} \cdot \mu_{j_2} \cdot \mu_{j_4}
\]
\[
D = \mu_{j_1} \cdot \mu_{j_3} \cdot \mu_{j_4}
\]
\[
E = \mu_{j_2} \cdot \mu_{j_3} \cdot \mu_{j_4}
\]
\[
A = \mu_{j_1} \cdot \mu_{j_2} \cdot \mu_{j_3} \cdot \mu_{j_4}
\]

In general:

\[
w_i = \frac{1}{1 - n + \sum_{j=1}^{n} \frac{1}{\mu_j}} = \frac{1}{S_i}
\]

(17)

The derivatives expression are:

\[
\frac{\partial w_i}{\partial \mu_j} = \frac{1}{\mu_j^2 \left(1 - n + \sum_{j=1}^{n} \frac{1}{\mu_j}\right)} = \frac{1}{\mu_j^2} \cdot \frac{1}{S_i} = \frac{1}{\mu_j^2} \cdot w_i
\]

(18)

if \(w_i > 0\).

3. When \(\nu \in (0, \infty]\), \(\nu \neq 1\), the activation degree of the \(i\)th rule with two input variables is

\[
w_i = i_H (\mu_{i_1}, \mu_{i_2}) = \frac{\mu_{i_1} \mu_{i_2}}{\nu + (1 - \nu)(\mu_{i_1} + \mu_{i_2} - \mu_{i_1} \mu_{i_2})}
\]

(19)

The aggregation operator in case of three inputs is

\[
w_i = i_H \left( \mu_{i_1}, \left( i_H \left( \mu_{i_2}, \mu_{i_3} \right) \right) \right) = i_H \left( \mu_{i_1}, \frac{\mu_{i_2} \mu_{i_3}}{\nu + (1 - \nu)(\mu_{i_2} + \mu_{i_3} - \mu_{i_2} \mu_{i_3})} \right)
\]

(20)

\[
= \frac{\mu_{i_1} \mu_{i_2} \mu_{i_3}}{\nu^2 (1 - \mu_{i_1})(1 - \mu_{i_2})(1 - \mu_{i_3}) + \nu(\mu_{i_1} + \mu_{i_2} + \mu_{i_3}) - M}
\]

Where

\[
M = (2\nu - 1)(\mu_{i_1} \mu_{i_2} + \mu_{i_1} \mu_{i_3} + \mu_{i_2} \mu_{i_3}) - (3\nu - 2)(\mu_{i_2} \mu_{i_3} \mu_{i_4})
\]

(21)

In general:

\[
w_i = \frac{1}{1 - \frac{1}{\nu} + \nu^2 \left( \frac{1}{\mu_{i_1}} + \frac{1}{\nu - 1} \right) \left( \frac{1}{\mu_{i_2}} + \frac{1}{\nu - 1} \right) \left( \frac{1}{\mu_{i_3}} + \frac{1}{\nu - 1} \right)}
\]

(22)

or \(w_i = \frac{1}{1 - \frac{1}{\nu} + P_i^v} \) when \(P_i = \nu^v - 1 \prod_{k=1}^{n} \left( \frac{1}{\mu_k} + \frac{1}{\nu - 1} \right)\)

(23)

The derivatives expression:

\[
\frac{\partial w_i}{\partial \mu_j} = \frac{P_i}{\mu_j \left(\nu^v - 1 \prod_{k=1}^{n} \left( \frac{1}{\mu_k} + \frac{1}{\nu - 1} \right) \right)^2} = \frac{P_i}{\mu_j \mu_j^2 - \mu_j^2} \cdot w_i^2
\]

(24)

If \(n \geq 2, \nu > 0\).

The system learning capability is affected by parameter value of the Hamacher t-norm. In the future, optimizing it we will improve the system function approximation performance.

For example, the character of Hamacher t-norm for some selected parameter values is illustrated in Figure 4.

![Figure 4. Graphs of Hamacher t-norm for some selected parameter values](image)
III. CONCLUSIONS

The original contributions of this paper are:

a) Extension of automatic fuzzy rule base identification. In Mamdani type inference system the Hamacher parametric and various non-parametric t-norms are proposed in order to reduce the system computational effort and processing time. The general Hamacher t-norm deployed as aggregation operator in the mentioned inference system with trapezoidal shaped fuzzy membership functions in fuzzy rule base identification is not investigated in our previous or other author’s papers.

b) The derivative expressions of various t-norms as aggregation operators in FRBI are given.

c) Comparing the graphical illustrations and examining the property of Hamacher t-norm we concluded that the parameter value essentially influences the graphs character which will affect the learning capability of the model identification system. We propose \( \nu \leq 3 \).

By extensive simulation experiments in the future we intend to define a quasi-optimal Hamacher operator interval

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