Determination of transport network topology 
influence of traffic characteristics

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Abstract – The transport system plays a prominent part for modern society. The studies in this area have an aim to improve the transport network by developing new optimization methods. This paper presents a new model to estimate the throughput system of the transport network nodes(junctions). The capacity analysis focuses on determining how the changes of the capacity value has influence on the transport network.

Index Terms – Capacity, Traffic, Transport network, Graph theory, Throughput system, Throughput

I. INTRODUCTION

The studies of transport systems play an essential role to support the basic functions of modern society. A transport system can be represented as a network. Traditionally, the transport network is analyzed in terms of traffic, which describe the network topology is represented by a set of nodes and links between them.

In this paper, we propose a new model for determining the throughput system/node (junction) to describe the traffic in a structure adapted to traffic optimization. The throughput system is a key factor in traffic characteristics determination (queue time, number of vehicles, speed).

II. PREVIOUS STUDY

In the society development, an essential element is represented by transport systems; their absence is unimaginable for our travel demand. The studies in this area are large and diverse, which led to the improvement of transport systems by developing new methods to optimize the infrastructure and propose different intelligent systems, that allows analysis of functionality and accessibility to the transport network.

According to [1], a public transport network consists of a large number of streets, interconnected roads which presents different models: star-shape, various forms of regular and irregular. By analyzing these models of a transport network are determined, obtained, recognized the features of road structures.

A technique that offers the facility of a structural analysis of road networks, such as connectivity, finding the shortest route, that way the cost, is graph theory that was used in [2-8].

In terms of a network of transport, graph theory represents a theoretical analysis model. Thus, a transport network contains a set of nodes that describe the locations in space and a set of links that shows different topological and geometric configurations. The topology itself refers to arrangement and connectivity of nodes and links of a network [15]. Using the topology, which can help us to determine a few numbers of parameters that characterize the transport network: the capacity of a node or link, speeds allowed, traffic concentrations.

In transport, the network capacity has a prominent part, being defined as the maximum number of vehicles crossing a node (junction) or a street segment in a time. Capacity is measured at each element of the network such as links (street segments) and as nodes (junction).

Previous research shows that the capacity of a street depends on several factors including geometric conditions, traffic conditions. Therefore, in transportation system capacity has become an important measure for planning and management of transport and also a topic research.

Over time, the measures have been proposed to improve network capacity. [9] studied road network capacity with the equilibrium assignment. This method is done to solve the equilibrium assignment problems, the results are not the real solutions. The modification in elastic demand user equilibrium problems is another approach to represent the network [10]. Later, [11-12] proposed a model to evaluate the network capacity and congestion effects and also travel behavior in route choice.


Capacity analysis attempts to provide a obvious understanding of traffic to facilitate the transport in a network.

In this article, the network geometry is define according to the theoretical approach which was previously presented. Using the graph theory gives us accessibility to performing an analytical study to calculate the maximum throughputs per nodes of the network. The network transport is influenced by outside sources of the transport system.

III. MODEL PROPOSED

The model proposed for the analysis of traffic in a transport network is built using elements of graph theory. For transport road, a node represents a junction/intersection, while a link may be assigned the elements characteristic of the street configuration such as: length-distance, capacity, speed between nodes (intersections) [16].

Thus, the topology of road networks are identified by a graph, which is represented by the probabilities matrix of transition. In this case, transitions represent the
connectivity mode; more precisely, specify the direction of the movement between the nodes of transport network and the probability assigned to each realized connectivity.

The model proposed the method to estimate the through relative capacity of a network node in a configuration of streets in the presence of the input sources of traffic and the output.

Therefore, the proposed model is:

\[
\lambda_{i,j} = \left[ S_{\text{input}} \right]_{i,N,i} \times \left[ \text{Flow}_{\text{input}} \right]_{i,N,i} + \left[ Q \right]_{i,N,i} \times \left[ \lambda \right]_{i,j} - \left[ S_{\text{output}} \right]_{i,N,i} \times \left[ \text{Flow}_{\text{output}} \right]_{i,N,i}
\]

Where:  
- \( N_i \) - number of input sources  
- \( N_{\text{No}} \) - number of output sources  
- \( N_{n} \) - nodes of transport network(junctions)  
- \( \lambda_{i,j} \) - relative throughput per node

To determine the relative throughput of a network node are used the matrices:

- The probabilities matrix of transition the \( N_i \) input sources to the network nodes \( N_{n} \), \( \left[ S_{\text{input}} \right]_{i,N,i} \), which describe the contribution of each source entry into the network at each node of the graph.
- The probabilities matrix of transition from the \( N_{n} \) network nodes to the output source \( N_{\text{No}} \), \( \left[ S_{\text{output}} \right]_{j,N_{\text{No}}} \), which describe the contribution of each node in the network for each output source.
- The probabilities matrix of transition between nodes of the road network modeled, \( \left[ Q \right]_{j,N_{n}} \), which describes the flow between the network nodes.

The matrix \( \left[ \text{Flow}_{\text{input}} \right]_{i,N,i} \) represents the contribution of input sources in the network, that contribution of the output sources network \( \left[ \text{Flow}_{\text{output}} \right]_{i,N,i} \).

This capacity calculation involves to solve a system of \( N \) equations with \( N \) unknowns.

The relative throughput determination offers the possibility to obtain the characteristic parameters of a transport network such as: vehicle flow (flow) for each junction, artery, the artery speed, concentration (density).

### IV. ANALYTICAL STUDY CASE

For this case study, we considered a graph consisting of three nodes (junctions), two sources of input and the output of the network, two. Network topology is based on the probabilities matrix of transition and is shown in Figure 1.

If we know that the probabilities matrix of transition from the sources \( N_{\text{SI}} \) to the \( N_{\text{NI}} \) network nodes is

\[
\left[ S_{\text{input}} \right]_{i,N_{\text{SI}}} = \begin{bmatrix} 0.2 & 0 \\ 0.8 & 0 \end{bmatrix}
\]

and that these sources have some contribution \( \left[ \text{Flow}_{\text{input}} \right]_{i,N_{\text{SI}}} = \begin{bmatrix} 32 \\ 44 \end{bmatrix} \);

the probabilities matrix of transition from the \( N_{\text{NI}} \) nodes to NO sources is

\[
\left[ S_{\text{output}} \right]_{j,N_{\text{NI}}} = \begin{bmatrix} 0.7 & 0 \\ 0.3 & 0.9 \end{bmatrix}
\]

and the associated contribution \( \left[ \text{Flow}_{\text{output}} \right]_{j,N_{\text{NI}}} = \begin{bmatrix} 30 \\ 76 \end{bmatrix} \), and the probabilities matrix of transition between nodes in the road network modeled is

\[
\left[ Q \right]_{j,N_{n}} = \begin{bmatrix} 0 & 0.2 & 0.6 \\ 0.7 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \end{bmatrix}
\]

be determinate the relative throughput per node using the proposed model.

The calculation of the throughput system involves to solve a system of equations:

\[
\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \\ 0.8 & 0 \end{bmatrix} \times \begin{bmatrix} 32 \\ 44 \end{bmatrix} + \begin{bmatrix} 0 & 0.2 & 0.6 \\ 0.7 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \end{bmatrix} \times \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}
\]

Solving this system of equations we obtain the maximum throughput per nodes/ junctions in the studied network:

\( \lambda_1 = 43.9028; \lambda_2 = 13.4722; \lambda_3 = 99.3611. \)
In the study case was done a several experiments. For each experiment was performed a new matrix of transition. The purpose of these experiments was to test and validate the model for real conditions. In table1, are given some of the transition probability matrix used for testing in the determination of the network topology.

Table 1. Network Topologies Results

<table>
<thead>
<tr>
<th>No.</th>
<th>Probabilities matrix of transition - Q</th>
<th>λ</th>
<th>Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.4 0.4]</td>
<td>(\lambda_1 = \infty) (\lambda_2 = \infty) (\lambda_3 = \infty)</td>
<td>a</td>
</tr>
<tr>
<td>2</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.3]</td>
<td>(\lambda_1 = 272.94) (\lambda_2 = 244.35) (\lambda_3 = 300)</td>
<td>b</td>
</tr>
<tr>
<td>3</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.2]</td>
<td>(\lambda_1 = 43.08) (\lambda_2 = 39.66) (\lambda_3 = 81.13)</td>
<td>d</td>
</tr>
<tr>
<td>4</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.3]</td>
<td>(\lambda_1 = 374.65) (\lambda_2 = 300) (\lambda_3 = 366.73)</td>
<td>c</td>
</tr>
<tr>
<td>5</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.3]</td>
<td>(\lambda_1 = 85.55) (\lambda_2 = 2.26) (\lambda_3 = 124.35)</td>
<td>d</td>
</tr>
<tr>
<td>6</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.3]</td>
<td>(\lambda_1 = 95.40) (\lambda_2 = 60.69) (\lambda_3 = 143.89)</td>
<td>d</td>
</tr>
<tr>
<td>7</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.3]</td>
<td>(\lambda_1 = 71.80) (\lambda_2 = 15.14) (\lambda_3 = 182.76)</td>
<td>d</td>
</tr>
<tr>
<td>8</td>
<td>[0.1 0.3 0.6] [0.7 0.2 0.1] [0.2 0.5 0.3]</td>
<td>(\lambda_1 = 37.40) (\lambda_2 = 33.80) (\lambda_3 = 296.40)</td>
<td>c</td>
</tr>
</tbody>
</table>

As you can see from Figure 2, where is define four types of network topologies, in which only one is functional in the real world. Thus, the first three topology are a clear method for testing the proposed model and the last one for validate the model.

a) The topology in which all the outputs of the network nodes to other nodes of the network have the sum of probabilities distributions equal to 1. In other words the output of nodes must be connected with outside nodes of the network. The model in which all the output nodes remain in the network has not a real application.

b) Another mode in which all the entries of the network nodes have the sum probability distributions equal to 1. This model has not a real application. And here some of the entries must be connected outside the network nodes.

c) A negative value for the throughput of a node defines the third network topology, which cannot be applied. We believe that this aspect has complex implications and we intend to study it in detail in another article.

d) The only topology that we found a correspondence in reality is a configuration which provides the positive values for the throughput. This topology is an accurate method of validation for the proposed model.

V. Model Validation

A. Java Modelling Tools

Java Modelling Tools (JMT) is an integrated environment for workload characterization and performance evaluation based on queuing models.[17]. JMT supports a wide range of activities which allows the performance evaluation, such as capacity planning model by simulation and analytical algorithms, workload characterization, automatic identification of bottlenecks in the network, modeling and optimization of analytical models. Usually, in JMT, the analysis of performance indices are focused on estimating such as throughput, the response time to requests, the degree of utilization.

For a successful validation of the proposed model, we have chosen the implementation in JMT and to use a model with open classes.
For a performance analyze we need to define a model. To define a model should be considered some steps such as: to define the customers classes, to define the parameters that describe the network topology, to define the performance indices. The models which are implemented must be defined by one or more classes of customers. For each customers class shall be established a reference source. The customers classes can be of two types: open and closed.

The model with open classes involves defining a parameter which describes the arrival rate of the customers in the system. The number of customers which arrive in the network is infinite. The number of customers can fluctuate in time. Customers that have finished their execution leave the system.

In model with closed classes is specified a parameter which describes the average number of customers in execution.

The proposed model cannot be implemented with the model with closed classes because a transport network never can be a closed system.

B. Validation of model with open classes.

To build the model was necessary to define the arrival rates for the two types of customers. For class 1 was considered a total of 32 customers per unit of time, and for class 2 a total of 44 customers per unit of time. We split all customers in two equal parts, one part for each class.

The graphical representation of the selected model can be viewed in Figure 4.

For each junction which was described in our model, we want to calculate the performance indices: throughput, utilization. The obtained values during the simulation can be seen in Table II.

The graphic evolution of these performance indices can be seen in figures 5 and 6.

![Figure 4. Model with open class](image)

![Figure 5. The throughput evolution for each node (from top to bottom: Node 1, Node 2 and Node 3.](image)

![Figure 6. The utilization evolution for each node (from top to bottom: Node 1, Node 2 and Node 3.](image)

<table>
<thead>
<tr>
<th>No.</th>
<th>Performance indices</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Throughput for node 1</td>
<td>0.0405</td>
</tr>
<tr>
<td>2</td>
<td>Throughput for node 2</td>
<td>0.0432</td>
</tr>
<tr>
<td>3</td>
<td>Throughput for node 3</td>
<td>0.0204</td>
</tr>
<tr>
<td>4</td>
<td>System throughput</td>
<td>0.0164</td>
</tr>
<tr>
<td>5</td>
<td>Utilization for node 1</td>
<td>0.8896</td>
</tr>
<tr>
<td>6</td>
<td>Utilization for node 2</td>
<td>0.2886</td>
</tr>
<tr>
<td>7</td>
<td>Utilization for node 3</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

After the simulation, we analyze the obtained values and we can say that these are included between the analytical model parameter. Because the system was not saturated we must conclude that validation is successful.

VI. CONCLUSIONS

In this paper, we propose a model to estimate the throughput system/node (junction) for optimizing the traffic of the transport network. The proposed model is
designed using the topology of transport network. This network is influenced by the presence of the external sources.

Following the achieved experiments, we define four types of network topologies (a, b, c, d). They have been obtained on the variation of the probabilities matrix of transition between the nodes of network. The proposed model was tested and validated by all tried topologies.

An interesting observation that resulted from the experiments is how to determine the maximum throughput of the nodes network. The throughput is obtained only if the network nodes tend to have only outputs to other nodes (links are not allowed to the same node).

The obtained model was validated by using the JMT application, and the obtained parameters after the simulation match the analytical model parameter interval.

We conclude that the proposed model is viable and open new opportunities for development in this area.

ACKNOWLEDGMENT

This work was partially supported by the strategic grant POSDRU 107/1.5/S/77265, inside POSDRU Romania 2007-2013 co-financed by the European Social Fund – Investing in People.

This work was partially supported by the strategic grant POSDRU/89/1.5/S/57649, Project ID 57649 (PERFORM-ERA), co-financed by the European Social Fund – Investing in People, within the Sectoral Operational Programme Human Resources Development 2007-2013.

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