Abstract—The present paper presents specific issues that arise in Photovoltaic Energy Conversion Systems, being focused on maximizing the output power. There is proposed a model for Photovoltaic PV panel external characteristics. The coordinates of maximum power point are determined and used for imposing the appropriate load resistance to maximize the provided power.

A comparison is made considering the classical control methods and the proposed optimized method, that prove better performances than methods based on output power measurement.

I. INTRODUCTION

Considering the renewable energy sources, it can be noticed that in the last years the solar electro-energetic systems are becoming increasingly used in obtaining electric energy [1, 5, 6]. Due to its geographical position, in certain areas of Romania, the usage of solar power systems is very efficient.

In the specialized literature focused on converting solar energy in electric energy based on photovoltaic (PV) effect [3, 4, 9, 10], the photovoltaic system is expected to operate at the maximum power points [11, 12]. The maximum power coordinates (voltage: \( U_{OPTIM} \), current: \( I_{OPTIM} \)) are changing in time, depending on meteorological conditions (solar radiation intensity) and consequently the equivalent load from PV module must be correlated with the solar radiation intensity [15, 17].

The PV panel external characteristic \( U = f(I) \) is varying according to the cloudiness of the atmosphere.

The power provided by PV panel varies significantly in function of the load current, having a maximum value for the current \( I^* \).

\[
P_{\text{max}} = U^* I^* \quad (1)
\]

The coordinates (\( U^* \) and \( I^* \)) of the maximum power point depends on solar radiation intensity [14, 16]. The PV control system must ensure the functioning at \( P_{\text{max}} \) since this value is constantly changing depending on the degree of sunburn (clouds, fog, pollution).

Therefore, the power received from the sun changes continuously and the PV energetic system (PVES), constituted from PV panel, power converter (DC-DC) and storage battery (SB), shall be such adjusted as to operate the maximum power point.

As is demonstrated below, controlling the PVES based on measured output power does not achieve a maximum capture of solar energy available. A more effective solution is one that is based on maximum power point coordinates: voltage \( U_{OPTIM} \) and current \( I_{OPTIM} \). In the respect of this idea the external characteristics of PV panel are modeled.

II. EXTERNAL CHARACTERISTICS MODELING

Based on experimental external characteristics [18], presented in figure 1, an analytical relation was determined, to approximate as good as possible the dependences \( U(I) \).

\[
U(I) = (d - T \cdot f) \left( \cos \left( \frac{a \cdot I - b \cdot T}{P_s} \right) \right)^g
\]

where: \( a, b, c, d, f \) and \( g \) are constructive constants, \( I \) is the output current. The constants \( a, b, c, d, f \) and \( g \) are determined from the experimental external characteristics.

The coordinates (voltage \( U_{OPTIM} \) and current \( I_{OPTIM} \)) of maximum power points \( P_{OPTIM} \), presented in figure 3, are determined considering the power derivate:
characteristic determination, for example. The dependences of the external characteristics change their shape, so they require a periodic reevaluation. The dependences $U(I)$ can be determined easily and accurately only experimentally for each PV panel in part or only the most important PV panels from the specific location.

Through periodical experimental determination of external characteristics $U(I)$, the value of maximum power is known, at any moment, being possible to adjust the load so that the solar energy captured by PV panel is maximum. By using this control method, a maximum energy is achieved at low cost and timely.

External characteristics experimental update depends on the location where PV panels operate, considering:
- pollution degree with dust deposits;
- weather changes during the day.

Knowing the dependence between voltage $U$ and current $I$ in an accurate manner is very important in determining the maximum power point. Experimentally, the problem is quite simple, considering a montage schema depicted in figure 2. Thus, the determination of two extreme operating points (presented in figure 3) is carried out as follows:

- **unload** (point A) – by opening $K_1$ and $K_2$ switches, results $U_0 = U_A$ and $I = 0$;
- **short circuit** (point B) by closing $K_1$ and $K_2$ switches, results $I_{SC} = I_B$ and $U = 0$.

![Figure 2. PES - Montage schema](image)

Four more points are needed to complete the external characteristic determination, for example $C$, $D$, $E$ and $F$. These operating points can be found by measuring the voltage and current at four values $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ of the conduction angle of the converter switching elements. All measurements have to be performed considering the PV panel temperature $T$ and a solar radiant power $P_s$.

Experimental determination of PV characteristics can be performed at different solar radiant power and temperature values, several times of the day, knowing very precisely the dependence $U(I)$, Thus, the maximum power point $P_{OPTIM}$ coordinates (voltage $U_{OPTIM}$, current $I_{OPTIM}$ and resistance $R_{OPTIM}$) are fully determined.

The time required for experimental measurements is low (mere seconds) and therefore energy loss due to system interruption is insignificant.

### B. Case study

The mathematical model (2), proposed for external characteristics $U(I) = f(I)$, can be determined based on experimental external characteristics [18] from figure 1. At $P_s = 1000 W/m^2$, $I = 0[A]$, $T = (273.15 + 25)[K]$ and $U_0 = 43[V]$ results: $d = -(273.15 + 25)f$ (6)

Similar, $P_s = 1000 W/m^2$, $I = 0[A]$, $T = (273.15 + 50)[K]$ and $U_0 = 39[V]$ results: $39 = d - (273.15 + 50)f$ (7)

solving (6) and (7) results $f = 0.16, d = 90.704$ (8)

At $P_s = 1000 W/m^2$, $I = 5.2[A]$, $T = (273.15 + 25)[K]$ and $U_0 = 0[V]$ results: $\frac{\cos{5.2}}{1000.4} = \frac{f}{2}$ (9)

At $P_s = 1000 W/m^2$, $I = 5.3[A]$, $T = (273.15 + 50)[K]$ and $U_0 = 0[V]$ results: $\frac{\cos{5.3}}{1000.4} = \frac{f}{2}$ (10)

For $I = 5[A], U = 35[V], T = (273.15 + 25)[K]$ results

$$35 = 43 \cos\left(\frac{\alpha_{5.2}}{1000.4}\right)$$

For $P_s = 800 W/m^2$, $T = (273.15 + 25)[K], I = 3.829[A]$, $U = 3647[V]$ and $3647 = 43 \cos\left(\frac{\alpha_{3.829}}{1000.4}\right)$ (12)

The solution of (9), (10), (11) and (12) is:

$$\alpha = 446.33 b = 1.056 g = 1.2315 \times 10^{-4}, c = 7.3737 \times 10^{-2}$$ (13)

Results the modeled external characteristic:
In the following there is considered a simplified mathematical model for the temperature $T = (298.15)$ K. The constants $a$, $b$, and $c$ of the proposed simplified mathematical model for external characteristics are determined based on the experimental external characteristics [18]: 

$$U(I) = 43\left(\cos\left(\frac{446.32}{I^{P_{\text{OPTIM}}}}\right)\right)^2 \times 10^{-2}$$  

(14)

$$U(I) = 43\left(\cos\left(\frac{446.32}{I^{P_{\text{OPTIM}}}}\right)\right)^2 \times 10^{-2}$$  

(15)

At $P_s = 1000[\text{W/m}^2]$, $I = 5.2[A]$ and $U = 0$ results:

$$\left(\frac{446.32}{1000}\right)^2 = \frac{c}{2}$$  

(16)

At $P_s = 500[\text{W/m}^2]$, $I = 2.5[A]$ and $U = 0$ results:

$$\left(\frac{446.32}{500}\right)^2 = \frac{c}{2}$$  

(17)

From (16) and (17) results $b = 1.0566, a = 446.32$ (19)

For $I = 5[A]$, $U = 35[V]$ results $c = 7.3737 \times 10^{-2}$ (20)

Consequently the modeled external characteristics (depicted in figure 4) are given by:

$$U(I) = 43\left(\cos\left(\frac{446.32}{I^{P_{\text{OPTIM}}}}\right)\right)^2 \times 10^{-2}$$  

(21)

![Figure 4. External characteristics modeled for $T=298.15$ K](image)

**Coordinates determination for maximum power points $P_{\text{OPTIM}}$**

Considering $P_s = 1000[\text{W/m}^2]$ and (21) results

$$P = U \cdot I = 43\left(\cos\left(\frac{446.32}{I^{P_{\text{OPTIM}}}}\right)\right)^2 \times 10^{-2} \cdot I$$  

(22)

And from

$$\frac{dP}{dI} = 43\left(\cos\left(\frac{446.32}{I^{P_{\text{OPTIM}}}}\right)\right)^2 \times 10^{-2} \cdot I = 0$$  

(23)

Results the current $I_{\text{OPTIM}} = 4.8472[A]$ (24) and corresponding voltage

$$U_{\text{OPTIM}} = 43\left(\cos\left(\frac{446.32}{1000}\right)\right)^2 \times 10^{-2} = 36.474[V]$$  

(25)

power $P_{\text{OPTIM}} = U \cdot I = 36.474 \cdot 4.8472 = 176.80[W]$ (26) and load resistance $R_{\text{OPTIM}} = \frac{U}{I} = 7.5248[\Omega]$ (27)

Considering $P_s = 800[\text{W/m}^2]$ and (21) results the current $I_{\text{OPTIM}} = 3.8291[A]$, voltage $U_{\text{OPTIM}} = 36.474[V]$, power $P_{\text{OPTIM}} = 139.66[W]$ and load resistance $R_{\text{OPTIM}} = 9.5255[\Omega]$.

Considering $P_s = 500[\text{W/m}^2]$ results the current $I_{\text{OPTIM}} = 2.3304[A]$, voltage $U_{\text{OPTIM}} = 36.473[V]$, power $P_{\text{OPTIM}} = 84.997[W]$ and load resistance $R_{\text{OPTIM}} = 15.651[\Omega]$. Also it can be noticed that:

$$P_{\text{OPTIM}} = 0.1768P_s$$  

(28)

Based on the obtained data, the between optimum power $P_{\text{OPTIM}}$, respectively optimum load resistance $R_{\text{OPTIM}}$ from solar radiant power $P_s$ (depicted in figure 5) and dependencies between optimum voltage $U_{\text{OPTIM}}$, respectively optimum current $I_{\text{OPTIM}}$ from solar radiant power $P_s$ (depicted in figure 6) can be built.

![Figure 5. Dependence between $P_{\text{OPTIM}}$ and $R_{\text{OPTIM}}$ from $P_s$](image)

$OPTIMP$, $OPTIMR$

![Figure 6. Dependence between $U_{\text{OPTIM}}$ and $I_{\text{OPTIM}}$ from $P_s$](image)

$OPTIMU$, $OPTIMI$

Analyzing the above results the following comments can be made:

1) The influence of solar radiant power $P_s$ over the optimal voltage $U_{\text{OPTIM}}$ is insignificant.
2) The optimum load resistance $R_{\text{OPTIM}}$ is significantly affected by the solar radiant power $P_s$, $R_{\text{OPTIM}}$ decreasing when $P_s$ increases.
3) The maximum output power $P_{\text{OPTIM}}$ depends linearly (with good approximation) by the solar radiant power $P_s$.
4) The optimum current $I_{\text{OPTIM}}$ changes linearly with the solar radiant power $P_s$. 
III. CONTROL STRUCTURES

In almost all scientific papers from the specialized literature [1,4,7,11], there are proposed control structures of PVES thus that maximum power is captured. These control structures are based on electric power measurement \( P = U I \) and the load resistance changes in order to obtain maximum power output. There are presented various control structures, considering a constant solar radiant power \( s_P = 1000\,[\text{W}] \).

A. Case 1- resistance changes in steps according to \( \Delta P \)

The control strategy is based on the output power. The power provided by the PV panel is measured from time to time and load resistance changes depending on

\[
\Delta P = P(k) - P(k-1)
\]

(29)

where \( P(k) \) represents the output power measured at time moment \( t_k \), and \( P(k-1) \) represents the power at \( t_{k-1} \).

For several steps the following results are obtained:

\[
R_{\text{OPTIM}} = 7.5248[\Omega]
\]

<table>
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<td>7.5424</td>
<td>7.7201</td>
<td>7.9066</td>
<td>8.0932</td>
</tr>
</tbody>
</table>

By changing the load resistance as

\[
R_k = R_{k-1} - K(P_k - P_{k-1})
\]

is obtained, the first phase, an approaching of the maximum power point, \( P_1 \rightarrow P_{\text{OPTIM}} \), then the operating point moves to \( P_2 \rightarrow P_3 \), the load resistance becoming higher.

\[
P_{\text{OPTIM}} = 176.8[\text{W}]
\]

Figure 7. Operating point movement for \( K=0.91512 \)

Therefore, the control structure of PVES, in this form, does not provide maximum power point operation.

To achieve a functioning close to the point of maximum power, the load resistance has to be changed to increase output power when power decreases.

By changing the constant \( K \) from \( K=0.91512 \) to \( K=0.1512 \) the following results were obtained:

\[
R_{\text{OPTIM}} = 7.5248[\Omega]
\]

<table>
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<td>7.7201</td>
<td>7.9066</td>
<td>8.0932</td>
<td>8.280</td>
</tr>
</tbody>
</table>

Figure 8. Operating point movement for \( K=0.1512 \)

In this case, the operating points are crowded between \( P_a \rightarrow P_b \), far from \( P_{\text{OPTIM}} \).

B. Case 2- resistance changes at intervals \( T \)

Load resistance change after a time \( T=5[\text{s}] \) as follows:

\[
R_k = R_{k-1} - K \cdot \Delta P - K \cdot \int_0^T \Delta P \cdot dt
\]

(31)

Initially, at \( t=0 \), the load resistance and measured power are:

\[
R(0) = 7.9[\Omega], P(0) = 175.66[\text{W}]
\]

(32)

Further values are obtained as follows:

\[
R[\Omega], P[\text{W}]
\]

resulting

\[
R_{\text{OPTIM}} = 7.5248[\Omega], P_{\text{OPTIM}} = 176.8[\text{W}]
\]

(33)

From the above results, depicted in Figure 9, one can be noticed the inefficiency of the classical control system, in the sense that it does not assure a maximum point operation, energy loss being about 0.6 %.

\[
\text{Figure 9. Optimal load resistance/output power}
\]

All control structures that seek the maximum power point, through output power measurements, does not achieve a maximal capitation of the solar energy available due to the fact that operates always below the maximum power point as a result of the fact that the load resistance from PV panel terminal changes in steps according to the measured electrical power and any time, and consequently \( R \neq R_{\text{OPTIM}} \).

Only using the method knowing the maximum power point can be an optimum for the purposes of capturing a maximum energy, because it can assure a load resistance \( R = R_{\text{OPTIM}} \).
IV. OPTIMAL CONTROL STRUCTURE

Seeking the maximum power point \( P_{\text{max}} \) is much more efficient if the load resistance \( R_L \) is changed taking into consideration boths voltage \( U(V) \) and current \( I(A) \) (36) and \( \Delta U = U_{\text{OPTIM}} - U_{\text{REAL}} \) and \( \Delta I = I_{\text{OPTIM}} - I_{\text{REAL}} \). If the control strategy considers only the voltage or the current, the following issues occur:

a) in the interval \( A \leftrightarrow P_{\text{OPTIM}} \) (see figure 3), \( U(V) \) does not vary significantly;

b) in the interval \( B \leftrightarrow P_{\text{OPTIM}} \), \( I(A) \) does not vary significantly.

The control structure contains two proportional elements having as input \( \Delta I \), respectively \( \Delta U \). Summing the two outputs is obtained:

\[
\Delta R = K_I \Delta U + K_I \Delta I
\]

(34)

Thus the load resistance changes according to:

\[
R(K) = R(K - 1) - \Delta R = R(K - 1) - (K_I \Delta U + K_I \Delta I)
\]

(35)

The maximum power coordinates \( P_{\text{OPTIM}} \) for solar radiant power \( P_{\text{R}} = 1000[W/m^2] \) are: current

\[
I_{\text{OPTIM}} = 4.8472[A], \quad \text{voltage } U_{\text{OPTIM}} = 36.474[V], \quad \text{power } P_{\text{OPTIM}} = 176.8[W]
\]

and optimal load resistance

\[
R_{\text{OPTIM}} = 7.5248[\Omega].
\]

Considering at \( t = 0 \) a load resistance \( R(0) = 8[\Omega] \) the control is performed in steps at different moments \( T \), the load resistance changing as follow:

\[
R(k) = R(k - 1) + K \cdot (U_{\text{OPTIM}} - U_{\text{REAL}}) - K \cdot (I_{\text{OPTIM}} - I_{\text{REAL}})
\]

(36)

The corresponding current error:

\[
\Delta I = I_{\text{OPTIM}} - I_{\text{REAL}} = 4.8472 - 4.6859 = 0.1613[A]
\]

(37)

And the voltage error:

\[
\Delta U = U_{\text{OPTIM}} - U_{\text{REAL}} = 36.474 - 37.487 = -0.1013[V]
\]

(38)

Since the optimum load resistance is \( R_{\text{OPTIM}} = 7.5248[\Omega] \) results:

\[
\Delta R = R_{\text{OPTIM}} - R_{\text{REAL}} = 7.5248 - 8 = -0.4752[\Omega]
\]

(39)

Considering (36) and (39) results

\[
0.4752 = -K \cdot \Delta U + K \cdot \Delta I = K \cdot 0.1013 + K \cdot 0.1613
\]

And the solution is \( K = 0.4 \). This is used to compute the necessary load resistance for the next step:

\[
R(1) = R(0) - \Delta R = 8 - (-0.4 \cdot \Delta U + K \cdot 
\Delta I) = 7.5303[\Omega]
\]

(40)

Step 1 - for \( t = 1s \):

Load resistance \( R(1) = 7.5303[\Omega] \), output current \( I = 4.8454[A] \) and from (21) results the PV voltage

\[
U(1) = 43 \cdot \cos \left( \frac{446.32 \cdot 4.8454}{1000 \cdot \cos^2(\theta)} \right) \cdot 10^{-5} = 36.478[V]
\]

and output power \( P(1) = 176.79[W] \)

The corresponding current error:

\[
\Delta I = I_{\text{OPTIM}} - I_{\text{REAL}} = 4.8472 - 4.8454 = 0.0018[A]
\]

(41)

And the voltage error:

\[
\Delta U = U_{\text{OPTIM}} - U_{\text{REAL}} = 36.474 - 36.487 = -0.0013[V]
\]

(42)

The computed necessary load resistance for the next step:

\[
R(2) = 7.5303 - 0.4(0.013 + 0.0018) = 7.5244[\Omega]
\]

(43)

Step 2 - for \( t = 2s \):

Load resistance \( R(2) = 7.5244[\Omega] \), output current \( I = 4.8473[A] \) and from (21) results the PV voltage

\[
U(2) = 43 \cdot \cos \left( \frac{446.32 \cdot 4.8473}{1000 \cdot \cos^2(\theta)} \right) \cdot 10^{-5} = 36.473[V]
\]

and output power \( P(2) = 36.473 \cdot 4.8473 = 176.8[W] \)

The corresponding current error:

\[
\Delta I = I_{\text{OPTIM}} - I_{\text{REAL}} = 4.8472 - 4.8473 = -0.0001[A]
\]

(44)

And the voltage error:

\[
\Delta U = U_{\text{OPTIM}} - U_{\text{REAL}} = 36.474 - 36.473 = 0.0001[V]
\]

(45)

The computed necessary load resistance for the next step:

\[
R(3) = 7.5244 - 0.4(-0.001 - 0.0001) = 7.5248[\Omega]
\]

(46)

Step 3 - for \( t = 3s \):

Load resistance \( R(3) = 7.5248[\Omega] \), output current \( I = 4.8472[A] \) and from (21) results the PV voltage

\[
U(3) = 36.474[V]
\]

and consequently the output power \( P(3) = 176.8[W] \).

The corresponding current error:

\[
\Delta I = I_{\text{OPTIM}} - I_{\text{REAL}} = 4.8472 - 4.8472 = 0[A]
\]

(47)

And the voltage error:

\[
\Delta U = U_{\text{OPTIM}} - U_{\text{REAL}} = 36.474 - 36.474 = 0[V]
\]

(48)

The computed necessary load resistance for the next step:

\[
R(4) = 7.5248 - 0.4(0) = 7.5248[\Omega] = R_{\text{OPTIM}}
\]

(49)

Figure 10. Control structure considering \( U_{\text{OPTIM}} \) and \( I_{\text{OPTIM}} \)
From the presented results, one can notice that after completion 3 steps, the operating point $P_3$ overlaps on $P_{\text{OPTIM}}$, a fact that proves the efficiency of the proposed control method.

V. CONCLUSIONS

Considering the presented simulations, there can be noticed the deficiencies related to classical control methods used in PVECS. These methods seek the maximum power point but operate below the optimal value, leading to significant power loss in case of PV panel of higher capacity.

Using the proposed control algorithm, based on the mathematical model of external characteristics, assure a functioning of PVES at optimum power point $P_{\text{OPTIM}}$ maximizing the harvested energy. These facts are possible through an accurate model of external characteristics (obtained through periodical experimental reevaluations) and knowledge about the maximum power point coordinates.

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