Brain Connectivity Measure – the Direct Transfer Function – Advantages and Weak Points

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Abstract— In this article we briefly investigate the Directed Transfer Function, the DTF, a causality measure used in determination of brain connectivity patterns, characterizing specific brain states/activities. Introduced by Kaminski and Blinowska two decades ago, it was claimed that this measure is the proper measure for complexity involved in brain activity, correcting incapacity and essential weakness of original Granger causality measures. This measure of brain connectivity reached broad application in very exciting brain activity modeling applied numerously by many research teams. Some of DTF advantages and weak points are analyzed in this article.

I. INTRODUCTION

Since the late sixties Granger’s method enabling detection of causality in time series [10], [11], [12], has been efficiently applied first in economy, stock market predictions, which lead to the Nobel price given to Granger in 2003. Rather early, it was understood that the Granger’s method can be applied to analysis of neurologic data, specifically to determine if there is a causality relationship between brain signals. Initial method affirmation was extended for this purpose by contribution of Geweke who greatly enriched the concepts and mathematics involved towards a complete study of the directed influence between two signal or two vectors of signals, their dependence in individual frequencies, relationship between time and frequency domain measures, establishing a sequence of directed connectivity measures, which could well express fine details in the connectivity investigations. Exciting developments followed. More than 20 years after original Granger paper, Kaminski and Blinowska introduced their Directed Transfer Function in the multivariate model, generalizing to some extent work of Granger, Geweke and the first followers, claiming superiority of their brain connectivity measure over original Granger and Geweke. Diverse other approaches to the problem of estimation of connectivity of brain signals are numerous and applied in specific circumstances [2-6] and [7-9].

The DTF was well affirmed and applied in many interpretations of experiments by its originators, their followers and other teams investigating brain connectivity. Recently, advanced measures with increasing potential are stepping in. The field of brain investigation completely changed with the orthogonal shift from attention on the individual centers as responsible for specific tasks, to identifying multitasking corresponding to the directed graphs connecting complexes of brain loci, the brain connectivity patterns involved in activation of individual neurologic functions. In rather short time numerous sensory and motoric functions are revealed in unprecedented richness. Brain Computer Interfaces, a mere science fiction less than 20 years ago became reality in fastly growing diverse applications, pushing numerous attractive challenges to broader Artificial Intelligence – AI field and expanding it unexpectedly towards multi brain multi computer efficient networking foreseen in the nearer future. The precisely detailed mapping of brain connectivity patterns related to cognitive tasks is realized by independent research teams.

We briefly present methods, models and connectivity measures involved, with the well established properties, then we proceed with our critical arguments, reproducing some of published data and results when necessary. Discussed methods are completely available in the cited and other literature.

II. METHODS

When there are three variables \( x(t) \), \( w(t) \) and \( y(t) \), if the value of \( x(t + 1) \) can be determined better using past values of all the three, rather than using only \( x \) and \( w \), then it is said that the variable \( y \) Granger causes \( x \), or \( G \)-causes \( x \). Here \( w \) is a parametric variable or a set of variables.

In the bivariate case, Granger causality is expressed using linear autoregressive model as

\[
x(t) = \sum_{j=1}^{p} a_{1j}x(t-j) + \sum_{j=1}^{p} a_{2j}y(t-j) + E_1(t) \\
y(t) = \sum_{j=1}^{p} a_{2j}x(t-j) + \sum_{j=1}^{p} a_{2j}y(t-j) + E_2(t),
\]

where \( p \) is the order of linear model and \( E_i \) are the prediction errors. The model consists of the linear recursive and the stochastic component. Thus, if coefficients of \( y \) in the first equation of (1) are not all zero, we say that \( y \) \( G \)-causes \( x \); similarly for variable \( y \).

First Geweke [13], [14] considered causality with sets or vectors of variables, later expanded to the multivariate case, e.g. [15] and others
\[
(1') \quad x(t) = \sum_{j=1}^{p} A(j)x(t-j) + E(t),
\]
where \(x(t) = (x_1(t), ..., x_n(t))\) is a vector of variables, \(A(j), j = 1, ..., p\) is a coefficient matrix defining variable contributions at step \(t-j\), \(E(t)\) are prediction errors. It is important to state that conditions on this model are that the covariances of variables are stationary; it is not easy, sometimes even rather hard to verify this condition.

The spectral form of G-causality (from (1') by Fourier transform), Geweke [13] has the form:

\[
(2) \quad A(\lambda)x(\lambda) = E(\lambda),
\]
where
\[
(3) \quad A(\lambda) = -\sum_{j=0}^{p} A(j)e^{-2i\pi\lambda j}, \quad \text{with} \quad A(0) = -I,
\]
or, solved for \(x\)

\[
(4) \quad x(\lambda) = A^{-1}(\lambda)E(\lambda) = H(\lambda)E(\lambda).
\]
Here \(H\) is a transfer matrix of the system.

Geweke [13] determined a number of directed measures for two blocks of variables. The G-causality measure from channel \(j\) to \(i\) at frequency \(\lambda\)

\[
(5) \quad t_{ij}^{2} = |H_{ij}(\lambda)|^2 = |a_{ij}(\lambda)|^2|A(\lambda)|^{-2}.
\]
The linear causality of \(y\) to \(x\) he defined as

\[
(6) \quad F_{y\rightarrow x} = \ln \left(\frac{\var{\epsilon_1}}{\var{E_1(t)}}\right), \quad \text{analogously for vector variables.}
\]

Geweke defined in the frequency domain the measure of linear causality at a given frequency, for two variables or two blocks of variables:

\[
(7) \quad F_{y\rightarrow x}(\lambda) = \ln \left(\frac{\var{S_{xx}(\lambda)}}{\var{S_{yy}(\lambda)}}\right), \quad H_{xx}^*(\lambda) = \text{the Hermitian transpose of } H_{xx}(\lambda),
\]
\(S_{xx}(\lambda)\) is the top left block of \(S(\lambda)\), the spectral density matrix

\[
S(\lambda) = \begin{bmatrix}
S_{xx}(\lambda) & S_{xy}(\lambda) \\
S_{yx}(\lambda) & S_{yy}(\lambda)
\end{bmatrix} = H(\lambda)S_{2}(\lambda)H^*(\lambda),
\]
\(H(\lambda) = \begin{bmatrix}
H_{xx}(\lambda) & H_{xy}(\lambda) \\
H_{yx}(\lambda) & H_{yy}(\lambda)
\end{bmatrix}\).

Geweke theoretical contribution does not stop here, but we suggest [13] as a more comprehensive exposition. With all this method developed, also in frequency domain, Kaminski and Blinowska [15] defined an adaptation of Granger-Geweke causality measure to \(m\) variables, with the same expression as in (5) which they called (non normalized) Directed Transfer Function (DTF); its normalization they defined as

\[
(8) \quad \text{DTF}_{ij}(\lambda) = |H_{ij}(\lambda)|\left(\sum_{k=1}^{n} |H_{ik}(\lambda)|^2\right)^{-1/2},
\]
measuring causality from \(i\) to \(j\) at frequency \(\lambda\). Clearly this approach is not a new invention – it is a minor generalization/reformulation of original Granger-Geweke work.

Probably major advance in Granger-Geweke framework was made by Baccala and Sameshima [18] who defined a normalized measure called Partial Directed Coherence (PDC), measuring direct influence of channel \(i\) to channel \(j\) at frequency \(\lambda\), with

\[
(9) \quad \text{PDC}_{ij}(\lambda) = \pi_{ij}(\lambda) = A_{ij}(\lambda)(a_{j}^*(\lambda)a_{i}(\lambda))^{-1/2},
\]
\(a_j\) is the \(j\)-th column in \(A(\lambda)\) and \(a_j^*\) is Hermitian transpose of \(a_j\). A variety of other connectivity measures were introduced by numerous researchers, many of those for specific purposes.

In numerous occasions and publications Blinowska and Kaminski claimed superiority of their measure to Granger (Granger-Geweke) measure insisting that DTF is exactly measuring directed connectivity between brain signals, especially in the case with multiple signal influences. In detailed studies using simple synthetic linear models with 5 to 7 signals, Bacalla and Sameshima have shown that DTF does not differentiate between directed influence of one signal to another and the transitive relationship of further nodes in the network of brain signals, e.g. [17], [18], [21], [22]. In the case of more complex – recorded brain signals, they proved the same DTF deficiencies. The authors of DTF tried to correct the problem by introduction of complementary used direct connectivity measure – DC, earlier introduced and exploited by various authors (e.g. [17]).

\[
(10) \quad \text{DC}_{ij}(\lambda) = \pi_{ij}H_{ij}(\lambda)(\sum_{k=1}^{n} \sigma_{ik}^2 |H_{ik}(\lambda)|^2)^{-1/2},
\]
\(\sigma_{kl}\) (\(k, l = 1, ..., n\)) are components of the covariance matrix \(\Sigma_{2}\).

We will discuss the DTF and other measures from the perspective of practical applications, when certain other moments are of increased importance. Among those is the estimation of statistical significance, which determines the fundamental zero threshold for each connectivity measure. This issue is rather sensitive and depends on multiple parameters and recording conditions. Rather broadly studied in the literature, e.g. [15-24], [30-31], for the above measures, the established significance in a variety of experiments, for PDC it is in the values from 0.01 and up, but with a little complexity, the renown authors determined it in the range from 0.05 to 0.20; for non normalized DTF it is 0.068 and for the normalized DTF the obtained value is 0.0045. We will skip the DTF performance on synthetic linear and simple nonlinear models, since this case is well elaborated in the cited literature as we put above.
As rather simple nontrivial example we will take a short, 2s experiment published in [18]. In cited literature there are other similar and different examples. Shortly, we have parallel recording of six brain areas with standard designation A10, A3, A17, CA1, CA3, and DG. Next, PDC and DTF calculations were performed which is shown in the Table 1, as frequency distributions, [0, 48] Hz, for each pair of signals (directed), for each of these measures. In each frequency distribution, maximum is determined for each pair of signals, for PDC, DTF respectively, which is shown in Table 1. These values are used in order to determine the connectivity diagrams for PDC and DTF, shown in Fig. 1. The common zero threshold for both measures was taken to be 0.2. The data, calculations and diagrams are reproduced from [18].

<table>
<thead>
<tr>
<th></th>
<th>A10</th>
<th>A3</th>
<th>A17</th>
<th>CA1</th>
<th>CA3</th>
<th>DG</th>
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Table 1. on the left, shaded spectra show PDC calculated connectivity matrix; on the right, shaded spectra show DTF connectivity matrix; DC is in solid lines. Reproduced from [18].

If we separate two properties: being connected in the shown direction as Conn predicate from connectivity degree, DegConn predicate we are in a position to analyse more essential first predicate independently. Then from the same above data we obtain the directed graph for each of the measures showing connectivity directions only, as presented on Fig 2.

However, if the calculations were performed with respect to above cited significance as thresholds, then its instability becomes clear, the DTF connectivity increases everywhere, completely changing corresponding connectivity structure, in consistence with the D. Adams axiom [32]

\[(∀x(∀y(Tresh(x, y) \rightarrow Conn(x, y)))),\]

(where Tresh predicate picks the elements beyond the threshold). This example presents the need for well argumented threshold determination when using DTF. It simply cannot be done arbitrarily. Together with already mentioned deficiency in distinguishing between directly and indirectly connected nodes in the connectivity graphs, this largely used method is in serious threat to produce wrong connectivity diagrams whenever there are cascades of connections. Consequently, a number of very interesting experimentation received wrong or non determined imprecision of interpretations. These findings are not singular, they are general, true for many published DTF based connectivity graph computations (e.g. all cited literature). As final examples we reproduce two sets of connectivity diagrams from very recent [31]. The number of similar published examples is large.

Table 2. the connectivity links are sorted column wise, i.e. in the first column are A10 links toward the areas defined as row names: each coordinate, on top shown is the PDC spectral maximum from the spectrum at corresponding coordinates in Table 1, below the spectral maximum for DTF. Reproduced from [18].

Figure 1. Connectivity diagrams relating activity of involved brain structures which are obtained from the matrices in Tab. 1 and Tab. 2 using: PDC for the left diagram, DTF for the right diagram. Diagrams involve five degrees of connectivity, weakest in dashed lines, increasing with the thickness of connection. Reproduced from [18].

Figure 2. The diagrams of difference in connectivity for the DTF and PDC, shown in Figure 1, for zero threshold equal to 0.20; this diagram is complementing graphs in Fig. 1: the union of this graph with the first in Fig. 1 becomes equal to the second in Fig.1 (with respect to Conn predicate only. Connectivity arrows show: DTF connected, while PDC is disconnected. We can see in this case that DTF connectivity is refining PDC connectivity, i.e. showing even connectivity links which are results of transitive influences which do not exist as real direct connections. This refinement is quite frequent but does not hold generally.
measure becomes unstable or non informative. The partial solutions we reached are partially available in [4-6].

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REFERENCES


