Abstract—In this paper a new algorithm for path tracking of robotic mechanism is presented. Through the history of robotic mechanisms, path tracking was considered one of the hardest tasks, due to many different problems. Usually, robotic mechanism is modeled as a rigid mechanism and that is simplified assumption. Elasticity of the mechanism needs to be considered and that represents one of the biggest problems. The example of the robotic mechanism examined in this paper has 6 DOF. Complexity of this mechanism comes from its elasticity and controlling it with two motors is a hard task. Presented method is based on controlling a mechanism with elastic parts to track desired path by following rotations of two motors.

Keywords—robotic mechanism; trajectory; elastic; rigid; modeling;

I. INTRODUCTION

The most usual example of robotic mechanism is a robotic arm. Every robotic mechanism, and therefore robotic arm, intend to simulate human actions. For successful imitation of human behavior, good modeling of robotic mechanism is essential. There are many papers and books related to this topic and the first one who introduced this problem was Spong [1]-[3].

Many applications, such as spray painting, plasma cutting and assembly, require good path tracking. Elasticity of robotic mechanism represents one of the biggest problems in path tracking. There are many papers dealing with modeling of robotic mechanism and its elasticity problems. In [4]-[8] authors present a new way of motor modeling and new form of mathematical model of robotic system in presence of elasticity elements. These researches are based on knowledge coming from [9]-[11].

This paper is organized as follows: In the first section two robotic mechanisms are defined. The first one is the rigid robotic mechanism, while the second one is elastic robotic mechanism and main goal is controlling the elastic robotic mechanism. In the third section trajectory algorithm and results are presented, while the final section shows conclusion.

II. ROBOTIC MECHANISM

As a starting step it is important to derive good models of two defined robotic mechanisms. First, model of the robotic mechanism with rigid parts (hereinafter referred to as the rigid robotic mechanism) is defined and afterwards a model of the robotic mechanism with elastic parts (hereinafter referred to as the elastic robotic mechanism).

Fig. 1 shows mechanism has two links and two motors that control them. Positioning of tip of the second link is controlled, so the chosen generalized coordinates are angles $\theta_1$ and $\theta_2$. $\theta_1$ represents angle between the $X_0$ axis and $X_1$ axis, while $\theta_2$ is angle between the $X_1$ axis and $X_2$ axis. In Fig. 1 it is shown that $(X_0,Y_0,Z_0)$ represents inner coordinate system in the starting of the first link, $(X_1,Y_1,Z_1)$ represents inner coordinate system of the first joint, $(X_2,Y_2,Z_2)$ represents inner coordinate system of the second joint and $(X_3,Y_3,Z_3)$ is coordinate system placed at the tip of the second link. First, it is important to determine connection between generalized and external coordinates. External coordinates represent $x$ and $y$ position of tip of the second link. From Fig. 1 it can be written:

\[
x = l_1 \cdot \cos(\theta_1) + l_2 \cdot \cos(\theta_1 + \theta_2) \quad (1)
\]

\[
y = l_1 \cdot \sin(\theta_1) + l_2 \cdot \sin(\theta_1 + \theta_2) \quad (2)
\]

From (1) and (2) it is important to find differentials of external coordinates, so it is written:

\[
x = -l_1 \cdot \dot{\theta}_1 \cdot \sin(\theta_1) - l_2 \cdot \sin(\theta_1 + \theta_2) \cdot (\dot{\theta}_1 + \dot{\theta}_2) \quad (3)
\]
\[
y = l_1 \cdot \dot{\theta}_1 \cdot \cos(\theta_1) + l_2 \cdot \cos(\theta_1 + \theta_2) \cdot (\dot{\theta}_1 + \dot{\theta}_2) \quad (4)
\]

After sorting (3) and (4) it can be written:
\[
x = -(l_1 \cdot \sin(\theta_1) + l_2 \cdot \sin(\theta_1 + \theta_2)) \cdot \dot{\theta}_1 - l_2 \cdot \sin(\theta_1 + \theta_2) \cdot \dot{\theta}_2 \quad (5)
\]

\[
y = +(l_1 \cdot \cos(\theta_1) + l_2 \cdot \cos(\theta_1 + \theta_2)) \cdot \dot{\theta}_1 + l_2 \cdot \cos(\theta_1 + \theta_2) \cdot \dot{\theta}_2 \quad (6)
\]

If \( X = [x \ y]^T \), then \( \dot{X} = [\dot{x} \ \dot{y}]^T \). Also, \( \theta = [\theta_1 \ \theta_2]^T \), then \( \dot{\theta} = [\ddot{\theta}_1 \ \ddot{\theta}_2]^T \). Now, using (5) and (6) it can be written:
\[
\dot{X} = J \cdot \dot{\theta} \quad (7)
\]

where \( J \) represents Jacobian matrix:
\[
\begin{bmatrix}
-l_1 \cdot \sin(\theta_1) + l_2 \cdot \sin(\theta_1 + \theta_2) & -l_2 \cdot \sin(\theta_1 + \theta_2) \\
+l_1 \cdot \cos(\theta_1) + l_2 \cdot \cos(\theta_1 + \theta_2) & +l_2 \cdot \cos(\theta_1 + \theta_2)
\end{bmatrix}
\]
(8)

Next step is deriving Denavit-Hartenberg parameters (DH parameters) at the starting moment. It is assumed that at the starting moment robotic mechanism lies on \( X \) axis and, therefore angles \( \theta_1 \) and \( \theta_2 \) are zero. DH parameters for this setting are shown in Table I:

<table>
<thead>
<tr>
<th>Segment</th>
<th>( q_i )</th>
<th>( a_i )</th>
<th>( d_i )</th>
<th>( \cos \theta_i )</th>
<th>( \sin \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>( l_1 )</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_2 )</td>
<td>0</td>
<td>( l_2 )</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

where \( l_1 \) is the length of first link, \( l_2 \) is the length of the second link, \( a_i \) presents angle between \( Z_i \) and \( Z_{i+1} \) axis, and as Fig. 1 shows these angles are always zero.

After conducting DH parameters, transform matrices are derived as:
\[
^0T_1 = \begin{bmatrix}
\cos \theta_1 & -\sin \theta_1 & 0 & l_1 \cdot \cos \theta_1 \\
\sin \theta_1 & \cos \theta_1 & 0 & l_1 \cdot \sin \theta_1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
^1T_2 = \begin{bmatrix}
\cos \theta_2 & -\sin \theta_2 & 0 & l_2 \cdot \cos \theta_2 \\
\sin \theta_2 & \cos \theta_2 & 0 & l_2 \cdot \sin \theta_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Total transform matrix is calculated as \( T = ^0T_1 \cdot ^1T_2 \).

Next step is deriving equations for kinetic and potential energy. Using Fig. 1 it can be written:
\[
E_k = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_2^2 + \frac{1}{2} I_1 (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} I_2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \quad (12)
\]

\[
E_p = m_1 g l_1 \sin \theta_1 + m_2 g l_2 \sin \theta_1 + m_2 g l_2 \sin (\theta_1 + \theta_2) \quad (13)
\]

Applying Lagrangian’s equations on (12) and (13) it can be written:
\[
\frac{\partial \dot{E}_k}{\partial \theta_1} = m_1 l_1 \dot{\theta}_1 + m_2 l_2 \dot{\theta}_1 + m_2 l_2 \dot{\theta}_2 + m_2 l_2 \sin (\theta_1 + \theta_2) \quad (14)
\]

\[
\frac{\partial \dot{E}_k}{\partial \theta_2} = m_2 l_2 \dot{\theta}_1 + m_2 l_2 \dot{\theta}_2 + m_2 l_2 \cos (\theta_1 + \theta_2) \quad (15)
\]

\[
\frac{\partial \dot{E}_p}{\partial \theta_1} = m_1 g l_1 \cos \theta_1 + m_2 g l_2 \cos \theta_1 + m_2 g l_2 \cos (\theta_1 + \theta_2) \quad (16)
\]

\[
\frac{\partial \dot{E}_p}{\partial \theta_2} = m_2 g l_2 \cos (\theta_1 + \theta_2) \quad (17)
\]

where \( m_{11} = m_1 + m_2 \) and \( J_{11} = J_{12} + J_{11} \). Using (14)-(17) matrix representation can be conducted:
\[
H \ddot{\theta} + h = \tau \quad (18)
\]

where \( H \) is inertia matrix:
\[
\begin{bmatrix}
m_1 l_1^2 + m_2 l_2^2 + m_2 l_2 J_{11} + J_{12} + J_{11} + 2 m_2 l_2 \cos \theta_1 & m_2 l_2 \cos \theta_1 & m_2 l_2 \cos (\theta_1 + \theta_2) \\
m_1 l_1 \cos \theta_1 & m_1 l_1 \cos \theta_1 & m_2 l_2 \cos (\theta_1 + \theta_2) \\
m_2 l_2 \cos \theta_1 & m_2 l_2 \cos (\theta_1 + \theta_2) & m_2 l_2 \cos (\theta_1 + \theta_2)
\end{bmatrix}
\]

(19)

and \( h \) is vector of Coriolis, Centrifugal and Gravitational forces, \( h = [h_1 \ h_2]^T \), where:
\[
h_1 = -m_2 l_2 \sin \theta_2 \cdot (\dot{\theta}_1^2 + 2 \dot{\theta}_1 \dot{\theta}_2) + m_2 g l_1 \cos \theta_1 \\

h_2 = m_2 l_2 \sin \theta_2 \cdot (\dot{\theta}_2^2) + m_2 g l_2 \cos (\theta_1 + \theta_2)
\]

(20)

(21)

and \( \tau = [\tau_1 \ \tau_2]^T \), where \( \tau_1 \) is load on the motor’s shaft. Motion equations of the motors are as follows:
\[
u_1 = R_1 l_1 + C_{E1} \dot{\theta}_1 \\
C_{M1} l_1 = I_1 \ddot{\theta}_1 + B_1 \dot{\theta}_1 + S_1 \tau_1
\]

(22)

\[
u_2 = R_2 l_2 + C_{E2} \dot{\theta}_2 \\
C_{M2} l_2 = I_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2 + S_2 \tau_2
\]

(23)

where \( R \) is the rotor motor resistance, \( L \) is the rotor current, \( C_{E1} = C_{E1} N_{V1}[V/(rad/s)] \) and \( C_{M1} = C_{M1} N_{M1}[Nm/A] \) are constants proportionality of electromotor force and torque, respectively, \( B \) is a viscous friction coefficient, \( I \) is the rotor and gearbox moment of inertia and \( S \) is a factor which defines geometry of gearbox.
B. Modeling of the elastic robotic mechanism

In [12] authors define dynamic model of flexible robotic system on classic principles of dynamics, but approach is more complex. Authors approach in realization of these principles with fact that elastic deformations are really presented and that they directly depend on immediate dynamic load of mechanism. Presented robotic mechanism is under the influence of environmental forces in the second part of the movement. In this paper, that part of the movement is not considered, because the goal is elimination of the elasticity effect. As it already was mentioned, robotic mechanism that has elastic parts is shown in Fig. 2.

Detailed modeling of the elastic robotic mechanism is presented in [12]. Final model of system with 6 DOF is as follows:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
H_{1,1} & H_{1,2} & H_{1,3} & H_{1,4} & 0 & 0 \\
H_{2,1} & 0 & H_{2,3} & 0 & 0 & 0 \\
H_{3,1} & 0 & 0 & H_{3,4} & 0 & 0 \\
H_{4,1} & 0 & H_{4,3} & 0 & H_{4,4} & 0 \\
0 & 0 & 0 & 0 & H_{5,5} & 0 \\
0 & 0 & 0 & 0 & 0 & H_{6,6}
\end{bmatrix} \begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3 \\
\dot{\theta}_4
\end{bmatrix}
\]

These equations are as follows:

\[
C_{M1}\dot{\theta}_1 = J_1\dot{\theta}_1 + B_1\dot{\theta}_1 - S_1(B_{13}\dot{\xi}_1 + C_{13}\xi_1)
\]

\[
u_1 = R_1\dot{t}_1 + C_{E1}\theta_1
\]

\[
C_{M2}\dot{\theta}_2 = J_2\dot{\theta}_2 + B_2\dot{\theta}_2 - S_2(B_{13}\dot{\xi}_2 + C_{13}\xi_2)
\]

\[
u_2 = R_2\dot{t}_2 + C_{E2}\theta_2
\]

III. TRACKING ALGORITHM AND RESULTS

In the second section two robotic mechanisms are defined. The elastic robotic mechanism shown in Fig. 2 is supposed to follow predefined trajectory, but due to the elasticity, this task is sufficiently difficult. Desired trajectory of this mechanism is shown in Fig. 4. It is assumed that mechanism should move from point A to point B in 2D space. Similar trajectory of the rigid robotic mechanism is shown in Fig. 3. The rigid robot mechanism has no problem with given task, while the elastic robotic mechanism has visible error. The main goal is to make elastic robotic mechanism to follow desired trajectory from point A to point B. It is important to notice that motors in both mechanisms are controlled with classical PD regulator.

A. The First example

Using model of the rigid robotic mechanism defined in the second section, referent trajectory is created. This trajectory is defined with angles \(\theta_1\) and \(\theta_2\) that change during the time of the mechanism’s movement. For easier representation these angles are labeled as \(\theta_{K1}\) and \(\theta_{K2}\). These angles are shown in Fig. 5.
After defining desired trajectory, the elastic robotic mechanism is controlled to perform a given task. As Fig. 2 and Fig. 4 show, relevant angles are $q_1$ and $q_2$. Considered variables are labeled as $q_{11}$ and $q_{12}$. These angles are shown in Fig. 6, together with $\theta_{K1}$ and $\theta_{K2}$. Fig. 7 presents difference between referent angles and simulated outputs of the elastic robotic mechanism.

### B. The Second example

As Fig. 6 and Fig. 7 show, mechanism with elastic parts has error and is not following desired trajectory. This error is defined as $e = [e_1 \ e_2]^T$, where:

\[
e_1 = \theta_{K1} - q_{11}, \ e_2 = \theta_{K2} - q_{12}.
\] (27)

Defined errors (Fig. 7) are present due to the elasticity of the mechanism, so these errors represent estimation of elasticity. For successful tracking, these errors must be removed, so the new referent trajectory is defined as follows:

\[
\theta_{N1} = \theta_{K1} - e_1, \ \theta_{N2} = \theta_{K2} - e_2.
\] (28)

The new referent trajectory is presented in Fig. 8, together with the first referent trajectory (desired). With this approach, when new referent trajectory is used for control of the elastic robotic mechanism, due to the elasticity it will not follow new referent trajectory, but it will follow...
the first referent trajectory (the rigid robotic mechanism) which represents the main goal. The output angles of the elastic robotic mechanism are labeled as $q_{21}$ and $q_{22}$. They are shown in Fig. 10 together with desired referent trajectory (the first one). Fig. 9 shows difference between the desired and output angles. As Fig. 9 and Fig. 10 show the difference between the desired trajectory and the output of the elastic robotic mechanism is small and mechanism follows referent trajectory.

IV. CONCLUSION

In this paper a new algorithm for path tracking of elastic robotic mechanism is presented. The example of the robotic mechanism examined in this paper has 6 DOF. Presented method is based on controlling a mechanism with elastic parts to track desired path by following rotations of two motors.

With defined approach, error due to the elasticity is used in good purpose and after applying new referent trajectory (28), the result is very similar to desired trajectory. Desired trajectory presents the one obtained with rigid robotic mechanism Desired trajectory is created to be smooth and there is no problem with tracking when there is no disturbance. By applying elasticity and by generating the trajectory using the method presented in this paper, trajectory is not smooth and it depends on elasticity parameters. Due to this problem a tracking error is present. Besides this problem, presented method is very precise and simple which is important in these applications, because of the possible use in more complicated systems, for an example, humanoids.

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