Iterative Learning Control Experimental Results for Inverted Pendulum Crane Mode Control


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Abstract—The paper deals with the application of two Iterative Learning Control (ILC) structures to the position control of the inverted pendulum system in crane mode. The two control system structures are based on serial and parallel ILC in combination with the conventional feedback control system structures with PI controllers. Recommendations to set the parameters of the ILC algorithms are given on the basis of experimental results.

I. INTRODUCTION

Iterative Learning Control (ILC) is based on the fact that the performance indices of control systems (CSs) executing repetitively the same tasks can be improved using previous experiments in the CS operation. The scope of ILC is represented by the iterative solving of a parametric optimization problem, referred to as learning, to ensure the minimization of an objective function specifying the CS performance indices [1]. ILC algorithms are numerical implementations of that solving. The CS performance enhancement from one experiment / iteration to another is ensured by the inclusion of information acquired from previous experiments making use of adequate memorizing techniques.

The inverted pendulum system (IPS) is an important benchmark that can illustrate several complex and nontrivial problems of control theory [2]. The IPS operating in crane mode is an industry inspired example of translational electromechanical system. Various solutions are reported in the literature in relation with IPS control [3]–[5]. The solutions concerning the position control of the IPS in crane mode concern also the gantry crane systems [6], [7]. The application of ILC to the control problems related to the inverted pendulum has been reported in [8]–[10], but accompanied only by digital simulation results.

In this context, this paper has two main contributions. First, a systematic application of the ILC to the position control of the IPS in crane mode is presented and two new control structures based on the combination of PI control and ILC are suggested. Second, real-time experimental results are included to validate the new CS structures. The conclusions are pointed out in Section V.

II. MODEL OF CONTROLLED PLANT AND CONTROL

PROBLEM SETTING

The experimental setup of the IPS is characterized by the block diagram presented in Fig. 1 [11], where: \( x_1 \) – the cart position (the distance from the center of the rail), \( x_2 \) – the angle between the upward vertical and the ray pointing at the center of mass cart (\( x_1 = 0 \) for the upright position of the pendulum), \( F \) – the control force, parallel to the rail, produced by the DC motor and applied to the cart, proportional to the PWM voltage signal (the control signal) \( u \) constrained to \( |u(t)| \leq u_{\text{max}} > 0 \), \( u_{\text{max}} = 0.5 \), and the cart velocity, \( m_c \) – the equivalent mass of the cart, \( m_p \) – the mass of the pendulum (pole + load), \( m_l = 0.052 \text{ kg} \), \( l \) – the distance from the axis of rotation to the center of mass, \( l = 0.011 \text{ m} \). Introducing the state variables \( x_3 \) (the cart velocity) and \( x_4 \) (the pendulum angular velocity) the state equations of the controlled plant (expressed in the absence of the independent time variable, \( t \)) are

\[
\begin{align*}
\dot{x}_1 &= x_3, \\
\dot{x}_2 &= x_4, \\
\dot{x}_3 &= \frac{1}{J_p}(m_c + m_p)\left[p \mu\left(m_c + m_p\right)l - x_2^2 \sin x_2 - (f_c - p_c)x_3\left(m_c + m_p\right)/l + \right. \\
&\left. + \left[g \sin x_2 - f_c x_3\left(m_c + m_p\right)/l\right] \cos x_2\right] \\
&\left./\left[J_p\left(m_c + m_p\right)/l^2 - \cos^2 x_2\right]\right], \\
\dot{x}_4 &= \frac{1}{J_p}\left[p \mu\left(m_c + m_p\right)/l - x_2^2 \sin x_2 - \\
&- (f_c - p_c)x_3\left(m_c + m_p\right)/l\right] \cos x_2 + \\
&+ \left[g \sin x_2 - f_c x_3\left(m_c + m_p\right)/l\right]/\left[J_p\left(m_c + m_p\right)/l^2 - \cos^2 x_2\right].
\end{align*}
\]
where: \( J_p \) – the moment of inertia of the IPS with respect to the axis of rotation, \( J_p = 0.00292 \, \text{kg} \cdot \text{m}^2 \), \( p_1 \) – the control force to PWM signal ratio, \( p_1 = 9.4 \, \text{N} \), \( p_2 \) – the control force to cart velocity ratio, \( p_2 = -0.548 \, \text{N} \cdot \text{s} / \text{m} \), defined in

\[
F = p_1 u + p_2 x_1, \tag{2}
\]

\( f_p \) and \( f_c \) – the rotational friction and dynamic cart coefficient, respectively, \( f_p = 6.65 \cdot 10^{-4} \, \text{N} \cdot \text{m} / \text{rad} \) and \( f_c = 0.5 \, \text{N} \cdot \text{s} / \text{m} \).

One typical CS structure that employs the cart and pendulum controllers with the transfer functions \( G_c(s) \) and \( G_p(s) \), respectively, is presented in Fig. 2 [12], where: \( u_1 \) and \( u_2 \) – the control signals elaborated by the two controllers, \( r_1 \) and \( r_2 \) – the reference inputs representing the desired cart position and pendulum angle, respectively, \( y_1 \) and \( y_2 \) – the controlled outputs i.e. the measured cart position and pendulum angle, respectively, \( e_1 = r_1 - y_1 \) and \( e_2 = r_2 - y_2 \) – the control errors, and \( d \) – the input vector that includes the disturbance inputs and the initial conditions. The controlled plant contains the sensors of the two controlled outputs and the interface corresponding to the actuator.

Two control problems representing the operating modes of the IPS are the crane mode and the self erecting mode illustrated in Fig. 3 [2], [11]. The reference inputs are set to \( r_2 = \pi \, \text{rad} \) for the crane mode and \( r_2 = 0 \, \text{rad} \) for the self erecting mode.

### III. OVERVIEW ON ITERATIVE LEARNING CONTROL

The controlled plant is considered to be characterized by the following discrete-time linear time-invariant Single Input-Single Output (SISO) system:

\[
y_j(k) = P(q)u_j(k) + d(k), \tag{3}
\]

where: \( d \) – the exogenous input signal (for example, load disturbance input) that repeats each iteration / trial, \( f \) – the index of the current iteration, \( g \) – the forward time-shift operator, \( P(q) \) – the proper rational transfer function of the plant, with the relative degree of \( m \in \mathbb{N}^+ \) and the delay of \( mT_s, T_s \) – the sampling period, \( k \) – the index of the current sampling interval.

The transfer function \( P(q) \) is supposed to be asymptotically stable. If not, the plant can be stabilized in the first phase, and the ILC is applied next to the stabilized CS.

Accepting the following sequences of \( N \) samples of inputs \( (u_j, d) \) and output \( (y_j) \) of the controlled plant:

\[
\{ (u_j(k), d(k)), k \in M_u \}, \quad \{ (y_j(k), k \in M_d \}, \quad M_u = \{0, 1, \ldots, N-1\}, \quad M_d = \{m, m+1, \ldots, N+m-1\},
\]

the control error is

\[
e_j(k) = r(k) - y_j(k), \quad r(k), k \in M_d. \tag{5}
\]

The popular Q-ILC algorithm [1] shows the feedforward character of ILC:

\[
u_{j+1}(k) = Q(q)[u_j(k) + L(q)e_j(k + 1)], \tag{6}
\]

where \( Q(q) \) is the Q-filter and \( L(q) \) is the learning function. The ILC algorithm (6) can be combined with conventional CSs employing feedback controllers to combine the advantages of both feedback and feedforward control. They lead to several CS structures including:

- the CS with serial ILC, where the control signal produced by the ILC algorithm, \( u_j(k) \), is added to the reference input before the feedback loop,

- the CS with parallel ILC, where \( u_j(k) \) is added to the control signal produced by the feedback controller.

The two CS structures can employ the simplified ILC algorithms (ILCAs) with PD-type learning functions:

\[
u_{j+1}(k) = Q(q)[u_j(k) + k_p e_j(k + 1) + k_d (e_j(k + 1) - e_j(k))], \tag{7}
\]

where \( k_p \) is the proportional gain and \( k_d \) is the derivative gain. Some theoretical aspects concerning the ILC are outlined in [1] and [13]. However the main requirements to ensure the improvement of the CS performance indices of the CS with ILCA in the real-time experiments are:
- The same initial conditions must be provided in all iterations.
- All iterations should have a finite length.
- The reference inputs should be defined within all iterations and they must be the same for all iterations.

IV. Control Structures and Real-Time Experimental Results

The proposed CS structures based on the combination of feedback control and ILC are presented in Fig. 4. They are dedicated to the crane mode control being extensions to the CS structure presented in Fig. 2. The two CS structures are referred to as CS with serial ILC, Fig. 4 (a), and CS with parallel ILC, Fig. 4 (b), where M stands for the memory block.

The frequency domain design has been applied to tune the cart controller with the transfer function \( G_c(s) \):

\[
G_c(s) = k_c (1 + T_i s) / s,
\]

(8)

where \( k_c \) is the controller gain and \( T_i \) is the integral time constant. First the controlled plant has been linearized at the operating point specific to the crane mode control problem. Next imposing the phase margin of 60° the parameters were tuned to the values \( k_c = 0.1 \) and \( T_i = 10 \) s.

The pendulum controller with the transfer function \( G_p(s) \) is not important because the controlled output \( y_2 \) has not been considered in order to simplify the controller design.

The Q-filter has been considered as a low-pass filter with the continuous-time transfer function \( Q(s) \):

\[
Q(s) = 1/(1 + T_f s),
\]

(9)

where \( T_f \) is the filtering time constant. The quasi-continuous digital implementation of the transfer functions in (8) and (9) has been applied making use of Tustin’s method for \( T_f = 0.01 \) s.

Several experiments have been done for the CS with serial ILC. The modification of the three parameters of the ILCA, \( k_p, k_d \) and \( T_i \), aims the improvement of the CS performance indices (overshoot and settling time) measured in the controlled output \( y_j = x_j \). Part of the real-time experimental results after 1 and 20 iterations is presented in Fig. 5 and Fig. 6.

Several aspects are highlighted in relation with the experiments done with the CS with serial ILC:

- The CS performance indices have been improved. However the improvement is not spectacular because the error is amplified in some experiments when the iteration index is increasing. The amplification appears because the injected signal is amplified by the controller resulting in large control signals. Therefore the controlled output is oscillating.

- For decreasing values of \( k_p \) and \( k_d \) the CS performance is acceptable but not very good. The Q-filter is extremely important with this regard. Large values of \( T_f \) lead to filtering all high frequency signals in the learning process. Thus the CS structure does not learn from too oscillatory signals.

- The small values of \( k_p \) and \( k_d \) result in large settling times. That has been shown in Fig. 6 with respect to Fig. 5 where smaller values of \( k_p \) and \( k_d \) lead to the alleviation of oscillations. In turn an increased settling time is observed.

- The process of setting the parameters of ILCA is not trivial, and the ranges of values that ensure the acceptable behavior of the CS are rather tight.
Part of the real-time experimental results for the CS with parallel ILC is presented in Fig. 7 and Fig. 8. The following aspects are outlined in relation with the experiments done with the CS with parallel ILC:

- The CS performance indices have been improved seriously depending on the parameter settings in the ILCA.
- The increased values of either $k_p$ or $k_d$ lead to larger overshoots and more oscillatory responses.
- The step-by-step increase of both $k_p$ and $k_d$ while keeping the constant value of $T_f$ is recommended.
- Once the values of $k_p$ and $k_d$ are set it is recommended to modify the values of $T_f$. They play a special role in the convergence of the ILCA and the overall CS performance indices.
- The influence of $T_f$ on the CS performance indices is more important for the CS structure with serial ILC.
- The ranges of values of the parameters of ILCA that ensure the acceptable behavior of the CS are larger for the CS structure with parallel ILC.
V. CONCLUSION

This paper validates by experimental results the application of ILC in combination with PI control in the crane control problem of to the inverted pendulum system. Two original control structures based on the serial and parallel ILC are given.

Useful recommendations are given for the practitioners. They are derived from the large number of real-time experiments. They can be generalized to controlling other nonlinear plants. Such generalizations should be accompanied by the analysis of the convergence of ILCAs in combination with feedback CS structures [14], [15].

One of the advantages of the ILC concerns the fact that the ILCAs can be used off-line although they rely on heavy computations. That makes them attractive for implementations on low-cost hardware, which is similar to the Iterative Feedback Tuning algorithms.
The first limitation of the ILCAs concerns the difficult analytical setting of the parameters. Many real-time experiments were needed in this paper to ensure the improvement of the CS performance indices. The second limitation deals with the assurance of the repeatability of the experience to make use of the advantages coming from the experience gained through previous experiments in the same conditions.

The limitations represent future research directions. The analytical tuning of the parameters of the ILCAs will be investigated starting with other plants that can be well approximated by linear / linearized models. Besides the real-time applications (design and implementation) of the ILCAs to other controlled plants (including nonlinear real-time applications (design and implementation) of the approximated by linear / linearized models. Besides the investigated starting with other plants that can be well analytical tuning of the parameters of the ILCAs will be the same conditions.

The experience gained through previous experiments in the the experience to make use of the advantages coming from the limitations deals with the assurance of the repeatability of improvement of the CS performance indices. The second experiments were needed in this paper to ensure the analytical setting of the parameters. Many real-time

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