2D fuzzy spatial relations and their applications to DICOM medical images

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Abstract—In this paper we propose a model of fuzzy spatial relations, which can be used to describe imprecise spatial data on 2D images. This model defines relations very similar to those existing in natural language and reasoning, like “on the left side of the image”, “in the center of the image”, “right/left of”, “above/below of”, “(very) close to”, “(very) far from”, “at distance of”, etc. Composition operators and, or, not and subtract are defined in order to combine these basic fuzzy spatial relations into more complex ones. In addition, formula for calculating membership value of imprecise region to fuzzy spatial relation is defined. Proposed model is verified through the example of practical application on DICOM medical images.

I. INTRODUCTION

Modeling of spatial data and using algorithms that process this data is often faced with two main problems. The first is related to the fact that boundaries between interior and exterior of object, or more generally, boundaries between objects are not accurately determined. The other problem is that modeling of spatial data relies on Euclidian space and geometry, which is an arithmetic with infinite precision. This is directly conflicted with the fact that computers are actually systems with finite precision [1]. For example, point of intersection of two lines must be rounded to the nearest coordinate that is in correspondence with image resolution. This inevitably causes not only numerical errors, but also topological errors. One approach to this problem is to actually accept this vagueness and extend existing models and algorithms in order to be able to process this data. Proposed approach is based on fuzzy logic and fuzzy sets theory [2]. Our previous work in [3],[4],[5],[6],[7] shows simple, yet efficient models of imprecise spatial data based on fuzzy sets.

Relations between spatial objects are an important aspect of the formal theory about spatial data. These relations were firstly formulated for ideal, crisp regions on images, not considering mentioned imperfection and imprecision of data. Because of this inherent property of digital images which means vagueness and imprecision, an alternative approach is needed. Thus, our approach considers extending classical methods of modeling spatial data and using fuzzy spatial relations.

Techniques and methods based on fuzzy logic and fuzzy sets have proven as very useful in medical image analysis. For example, in many cases of radiology practice, automated and intelligent image analysis and interpretation is considered as one of key parts of diagnoses procedure. In many cases, the most important step is image segmentation and region detecting (in medicine these regions are often referred to as “masses”). These masses are characterized by their position, size, shape, texture, etc. Because of huge variations in size and shape in which masses can appear, poor contrast of certain regions (masses), and generally because of imprecise image data [8], we propose using fuzzy spatial relations to model this data. Spatial layout of objects on images provides important information for tasks such as detection and interpretation, especially when objects are in complex environment, e.g. in medical images. Relations between objects can be partially described with terms like “left of”, “above”, “at distance of 20cm”, “in center of the image”, etc. However, it is important to notice that these concepts can often be ambiguous and hardly precisely defined, even they are completely natural and intuitive to human beings. It is clear that approach “true or false” leads to unsatisfactory results in many situations [9]. Thus, one solution is to rely on fuzzy logic and fuzzy relations. Fuzzy approach becomes even more interesting when image imprecision is taken in account, and it has many advantages [10]:

- it allows representation of imprecision which is inherent in the definition of a concept,
- it allows managing of imprecision that is related to expert knowledge in the concerned domain,
- it constitutes an adequate framework for knowledge representation and reasoning, reducing the semantic gap between symbolic concepts and numerical information.

This paper consists of six sections. Following this introductory section, definitions and model of fuzzy spatial relations are presented. In third section, composition operators of fuzzy spatial relations are described. Formula for calculating membership value of region to fuzzy spatial relation is given in section four.
Section five contains examples of practical application of the proposed model to DICOM medical images. The final section contains concluding remarks and future research directions.

II. Fuzzy Spatial Relations Model

Definition 1. Let $P$ be a two-dimensional matrix which represents digital image. For the fuzzy set $F$ we say it is fuzzy spatial relation if its membership function $\mu_f$ maps every element $P[i,j]$ of the matrix $P$ in the interval $[0,1]$

$$\mu_f(P[i,j]): P \rightarrow [0,1]$$

Basic concepts of fuzzy spatial relations are:

- **SpatialRelation** – fuzzy spatial relation.
- **SpatialObject** – fuzzy spatial object (in this solution it is a fuzzy polygon in linear fuzzy space presented in [7]).
- **ReferenceSystem** – reference system in which spatial relation is described (one spatial relation can be described in many ways, depending on the view perspective).
- **TargetObject** – instance of the SpatialObject concept. Represents the object for which spatial relation is determined with respect to some ReferenceObject.
- **ReferenceObject** – instance of the SpatialObject concept.
- **DirectionalRelation** – concept that extends SpatialRelation concept. Allows the modeling of unary spatial relation that represents position of a target object on the image.
- **BinarySpatialRelation** – concept that extends SpatialRelation concept. Allows the modeling of binary spatial relations between reference and target object.
- **BinaryDirectionalRelation** – concept that extends BinarySpatialRelation. Allows the modeling of position of target object with respect to reference object, e.g. “left of”, “above”, etc.
- **DistanceRelation** – concept that extends BinarySpatialRelation. Allows the modeling of distance of target object from reference object, e.g. “(very) close to”, “at distance of...”, etc.

After presenting basic concepts of fuzzy spatial relations model, we can define fuzzy spatial relations.

Remark. Coordinates $i$ and $j$ unambiguously determine every pixel of the image, where $i$ coordinate corresponds to position of pixel on horizontal axis, and $j$ coordinate to position of pixel on vertical axis of the image.

The definitions of the DirectionalRelation concept fuzzy spatial relations follow.

**Definition 2.** Let $P$ be a two-dimensional matrix that represents digital image, whose width and height are $w$ and $h$, respectively. If every pixel $P[i,j]$ is determined by coordinates $i$ and $j$, then the membership function value of fuzzy spatial relation $AtLeft$ (“at left side of the image”) is calculated as:

$$\mu_{AtLeft_{d,g}}(P[i,j]) = \begin{cases} 
1, & \text{if } i \leq d \\
1 - (i - d)/g, & \text{if } d < i \leq d + g \\
0, & \text{if } i > d + g.
\end{cases} \quad (1)$$

where $d$ is a constant which defines the core of this fuzzy set, and $g$ is a constant that defines the fuzziness of this fuzzy set. Graphical representation of the $AtLeft$ fuzzy spatial relation’s membership function is shown in Fig. 1.

![Figure 1 - Membership function of fuzzy spatial relation AtLeft](image1)

**Definition 3.** Let $P$ be a two-dimensional matrix that represents digital image, whose width and height are $w$ and $h$, respectively. If every pixel $P[i,j]$ is determined by coordinates $i$ and $j$, then the $AtBottom$ (“at bottom of the image”) fuzzy spatial relation’s membership function value is calculated as:

$$\mu_{AtBottom_{d,g}}(P[i,j]) = \begin{cases} 
1, & \text{if } j < d - g \\
(j - (d - g))/g, & \text{if } d - g \leq j < d \\
0, & \text{if } j \geq d.
\end{cases} \quad (2)$$

where $d$ is a constant which defines the core of this fuzzy set, and $g$ is a constant that defines the fuzziness of this fuzzy set. Graphical representation of fuzzy spatial relation $AtBottom$ is shown in Fig. 2.

![Figure 2 - Membership function of fuzzy spatial relation AtBottom](image2)

In a very similar manner, fuzzy spatial relations $AtRight$ and $AtTop$ are defined, and these shall not be shown in this paper.

**Definition 4.** Let $P$ be a two-dimensional matrix that represents digital image, whose width and height are $w$...
and \( h \), respectively. If every pixel \( P[i,j] \) is determined by coordinates \( i \) and \( j \), then the membership function value of fuzzy spatial relation \( AtCenter \) ("at center of the image") is calculated as:

\[
\mu_{AtCenter,d,g}(P[i,j]) = \begin{cases} 
1, & \text{if } i^2 + j^2 \leq d^2 \\
1 - \frac{1}{l - (l - d)g}, & \text{if } d^2 < i^2 + j^2 \leq (d + g)^2 \\
0, & \text{if } i^2 + j^2 \geq (d + g)^2.
\end{cases}
\]  

(3)

where \( l \) is distance of pixel \( P[i,j] \) from the center of the image, \( d \) is a constant which defines the core of this fuzzy set, and \( g \) is a constant that defines the fuzziness of this fuzzy set.

**Remark.** For more intuitive representation of the membership functions, grayscale images are used. Fuzzy set that defines fuzzy spatial relations is actually a two-dimensional matrix where for each pixel a value of membership function is calculated. The values from the real interval \([0,1]\) are then scaled to integer interval \([0,255]\) which represents gray color intensity. White color means maximal or full membership, black means minimal or no membership, and shades of gray mean partial membership.

Examples of \( AtTop \) and \( AtCenter \) fuzzy spatial relations are shown in Fig. 3.

![Figure 3 - Examples of a) AtTop and b) AtCenter fuzzy spatial relations](image)

The definitions of the BinaryDirectionalRelation concept fuzzy spatial relations follow.

**Definition 5.** Let \( P \) be a two-dimensional matrix that represents digital image, whose width and height are \( w \) and \( h \), respectively. Let \( \tilde{p} \) be a linear fuzzy polygon in a linear fuzzy space. For the polygon \( \tilde{p} \) that represents reference object, we define two points: \( maxTop \) (polygon point with smallest \( j \) coordinate), \( maxBottom \) (polygon point with largest \( j \) coordinate). If every pixel \( P[i,j] \) is determined by coordinates \( i \) and \( j \), then the membership function value of the fuzzy spatial relation \( RightOf \) ("right of") is calculated as:

\[
\mu_{RightOf,d,g}(P[i,j]) = \begin{cases} 
1, & \text{if } j_{maxTop} \leq j \leq j_{maxBottom}, i > right(i,\tilde{p}) \\
1 - \frac{l - \cos^{-1}(\theta)}{\pi}, & \text{if } l_{maxTop} < i < l_{maxBottom}, or \ i > l_{maxRight}, i > right(i,\tilde{p}) \\
0, & \text{otherwise},
\end{cases}
\]  

(4)

where function \( right(l, \tilde{p}) \) returns maximal value of \( j \) coordinate from points of polygon \( \tilde{p} \) whose \( j \) coordinates are equal to \( i \) coordinate of pixel \( P[i,j] \). Value \( \theta \) represents an angle which \( P[i,j] \) forms with point \( maxTop \) (if \( j < j_{maxTop} \)), or point \( maxBottom \) (if \( j > j_{maxBottom} \)), and constant \( g \) represents fuzziness of this fuzzy set.

**Definition 6.** Let \( P \) be a two-dimensional matrix that represents digital image, whose width and height are \( w \) and \( h \), respectively. Let \( \tilde{p} \) be a linear fuzzy polygon in a linear fuzzy space. For the polygon \( \tilde{p} \) that represents reference object, we define two points: \( maxLeft \) (polygon point with smallest \( i \) coordinate), \( maxRight \) (polygon point with largest \( i \) coordinate). If every pixel \( P[i,j] \) is determined by coordinates \( i \) and \( j \), then the membership function value of the fuzzy spatial relation \( Above \) ("above") is calculated as:

\[
\mu_{Above,d,g}(P[i,j]) = \begin{cases} 
1, & \text{if } l_{maxLeft} \leq l < l_{maxRight}, \ or \ i > l_{maxLeft}, i > Above(i,\tilde{p}) \ \\
1 - \frac{g}{\pi}, & \text{if } l_{maxLeft} < l < l_{maxRight}, or \ j > l_{maxRight} > Above(i,\tilde{p}) \\
0, & \text{otherwise},
\end{cases}
\]  

(5)

where function \( above(l, \tilde{p}) \) returns maximal value of \( j \) coordinate from points of polygon \( \tilde{p} \) whose \( i \) coordinates are equal to \( i \) coordinate of pixel \( P[i,j] \). Value \( \theta \) represents an angle which \( P[i,j] \) forms with point \( maxLeft \) (if \( i < l_{maxLeft} \)), or point \( maxRight \) (if \( i > l_{maxRight} \)), and constant \( g \) represents fuzziness of this fuzzy set.

In very similar manner, fuzzy spatial relations \( LeftOf \) and \( Below \) are defined, and these shall not be shown in this paper.

Examples of \( RightOf \) and \( Below \) fuzzy spatial relations with respect to an arbitrary reference object are shown in Fig. 4.

![Figure 4 - Examples of a) RightOf and b) Below fuzzy spatial relations](image)

The definitions of DistanceRelation concept fuzzy spatial relations follow.

**Definition 7.** Let \( P \) be a two-dimensional matrix that represents digital image, whose width and height are \( w \) and \( h \), respectively. Let \( \tilde{p} \) be a linear fuzzy polygon in a linear fuzzy space. For every pixel \( P[i,j] \) is determined by coordinates \( i \) and \( j \), and \( l \) is a distance between \( P[i,j] \) and reference object \( \tilde{p} \), then the membership function value of the fuzzy spatial relation \( (Very)CloseTo \) ("(very) close to") is calculated as:

\[
\mu_{CloseTo,d,g}(P[i,j]) = \begin{cases} 
1, & \text{if } l \leq d \\
1 - \frac{l - d}{g}, & \text{if } d < l \leq d + g \\
0, & \text{if } l > d + g.
\end{cases}
\]  

(6)
where \( d \) is a constant that defines the core of fuzzy set, and \( g \) is a constant that defines fuzziness of this fuzzy set. Values of \( d \) and \( g \) for relation \text{VeryCloseTo} are smaller than for relation \text{CloseTo}. Graphical representation of these relations is shown in Fig. 5.

**Definition 8.** Let \( P \) be a two-dimensional matrix that represents digital image, whose width and height are \( w \) and \( h \), respectively. Let \( \tilde{P} \) be a linear fuzzy polygon in a linear fuzzy space. If every pixel \( P[i,j] \) is determined by coordinates \( i \) and \( j \), and \( l \) is distance between \( P[i,j] \) and reference object \( \tilde{P} \), then membership function value of fuzzy spatial relation \text{(Very)FarFrom} ("(very) far from") is calculated as:

\[
\mu_{\text{FarFrom},d,g}(\tilde{P},P[i,j],l) = \begin{cases} 
1, & \text{if } r - d < l \leq r + d \\
\frac{l - (r - d)}{g}, & \text{if } r - d < l \leq r + d + g \\
\frac{l - (r - d - g)}{g}, & \text{if } r - d - g < l \leq r - d \\
0, & \text{otherwise},
\end{cases}
\]

where \( r \) is parameter of the relation and represents desired distance from the reference object. Constant \( d \) defines the core of fuzzy set (allowed deviation from desired distance), a constant \( g \) defines the fuzziness of fuzzy set. Graphical representation of relation \text{AtDistanceOf} is shown in Fig. 7.

**Example of \text{CloseTo}, \text{FarFrom} and \text{AtDistanceOf} fuzzy spatial relations with respect to an arbitrary reference object are shown in Fig. 8.**

**III. COMPOSITION OF FUZZY SPATIAL RELATIONS**

Elementary fuzzy spatial relations, as defined in section II, can be combined with use of operators that are introduced in this section, thus obtaining more complex fuzzy spatial relations. Defined operators are \text{AND}, \text{OR}, \text{NOT}, \text{SUBTRACT}.

**Definition 10.** Let \( F \) and \( G \) be two elementary fuzzy spatial relations. Than \text{AND} (conjunction) is a binary operator that performs composition of relations \( F \) and \( G \) in such way that value of membership function for each pixel \( P[i,j] \) is calculated as minimum of membership function values of \( F \) and \( G \) for pixel \( P[i,j] \):
An example of conjunction of two fuzzy spatial relations is shown in Fig. 9.

Figure 9 - Conjunction of two fuzzy spatial relations

**Definition 11.** Let $F$ and $G$ be two elementary fuzzy spatial relations. Then $OR$ (disjunction) is a binary operator that performs composition of relations $F$ and $G$ in such way that value of membership function for each pixel $P[i,j]$ is calculated as maximum of membership function values of $F$ and $G$ for pixel $P[i,j]$:

$$OR(\mu_F(P[i,j]), \mu_G(P[i,j])) = \max(\mu_F(P[i,j]), \mu_G(P[i,j]))$$

An example of disjunction of two fuzzy spatial relations is shown in Fig. 10.

Figure 10 - Disjunction of two fuzzy spatial relations

**Definition 12.** Let $F$ be an elementary fuzzy spatial relation. Then $NOT$ (complement) is an unary operator that behaves in such way that value of membership function for each pixel $P[i,j]$ is calculated as complementary of membership function value of $F$ for pixel $P[i,j]$:

$$NOT(\mu_F(P[i,j])) = 1 - \mu_F(P[i,j])$$

An example of complement of fuzzy spatial relations is shown in Fig. 11.

Figure 11 - Complement of fuzzy spatial relation

**Definition 13.** Let $F$ and $G$ be two elementary fuzzy spatial relations. Then $SUBTRACT$ (subtraction) is a binary operator that performs composition of relations $F$ and $G$ in such way that value of membership function for each pixel $P[i,j]$ is calculated as minimum of membership function value of $F$ and complementary membership function value of $G$ for pixel $P[i,j]$:

$$SUBTRACT(\mu_F(P[i,j]), \mu_G(P[i,j])) = \min(\mu_F(P[i,j]), 1 - \mu_G(P[i,j]))$$

An example of subtraction of two fuzzy spatial relations is shown in Fig. 12.

Figure 12 - Subtraction of two fuzzy spatial relations

IV. MEMBERSHIP OF LINEAR FUZZY POLYGON TO FUZZY SPATIAL RELATION

To this point, membership functions of fuzzy spatial relations and their compositions were defined. For each pixel on the image value of these membership functions were calculated. In addition, there is a need to find a measure in which some fuzzy polygon (effectively region on the image) “belongs” to some defined fuzzy spatial relation.

**Definition 14.** Let $\tilde{P}$ be a linear fuzzy polygon in a linear fuzzy space and $F$ be a fuzzy spatial relation defined with membership function $\mu_F$. If $P[i,j]$ is an element of a two-dimensional matrix $P$ that represents digital image $P$ whose width and height are $w$ and $h$, respectively, and if $\mu_F$ is a membership function which defines a linear fuzzy polygon $\tilde{P}$, then the membership of the linear fuzzy polygon $\tilde{P}$ to fuzzy spatial relation $F$ is calculated as:

$$\mu_F(\tilde{P}) = \frac{\sum_{i=0}^{w-1} \sum_{j=0}^{h-1} \mu_F(P[i,j]) \cdot \mu_F(P[i,j])}{\sum_{i=0}^{w-1} \sum_{j=0}^{h-1} \mu_F(P[i,j])}$$

V. AN EXAMPLE OF PRACTICAL APPLICATION

The proposed model of fuzzy spatial relations is used for determining spatial relations of “masses” on DICOM medical images. First, image segmentation and feature extraction algorithm [7] is applied to extract relevant regions. These regions are modeled as linear fuzzy polygons in a linear fuzzy space. Fig. 13a) shows a CT scan image, while Fig. 13b) shows the regions R1, R2 and R3 extracted from the scan image.

Figure 13 – a) CT scan and b) extracted regions

From the image 13b) based on visual observations we have created a linguistic description of the topological relations between extracted regions for the target object R2 in a following way: R2 is right of R1, and R2 is far...
but not very far from R1. R2 is also above R3 and at the same time it is not close to nor very far from R3.

So, the fuzzy relation G describing our observations can be expressed verbally as: “right of and far from R1, but not very far from R1, and above R3, but not close to nor very far from R3”.

This relation is expressed formally in our model as $G = (\text{RightOf}(R1) \text{ AND } \text{Above}(R3) \text{ AND } \text{FarFrom}(R1) \text{ AND } \text{NOT}(\text{VeryFarFrom}(R1)) \text{ AND } \text{NOT}(\text{CloseTo}(R3)) \text{ AND } \text{NOT}(\text{VeryFarFrom}(R3)))$.

Fig. 14 shows an illustrative representation of the fuzzy spatial relation G, and also obvious affiliation of the region R2 with this relation.

Calculated membership value of the region R2 to the relation defined G is $\mu_G(R2) = 0.78$ approves that the fuzzy spatial relation G describes the position of the region R2 appropriately in correspondence with visual observations.

CONCLUSION

In this paper we have proposed a model of fuzzy spatial relations which can be used to model imprecise spatial data. In order to model complex spatial data, one can combine more fuzzy spatial relations with certain composition operators. Future research direction might consider obtaining some kind of expert database with defined relations between relevant regions on medical images, in order to automate detection of these regions. Another direction is further development of the model, extending it to three-dimensional space and three-dimensional fuzzy spatial relations.

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REFERENCES