Application of Granular Computing and Three-way decisions to Analysis of Competing Hypotheses

Giuseppe D’Aniello*, Angelo Gaeta*, Matteo Gaeta†, Vincenzo Loia‡ and Marek Z. Reformat†

*Dip. di Ingegneria dell’Informazione, Ingegneria Elettrica e Matematica Applicata, University of Salerno, Fisciano, Italy
Email: gidaniello@unisa.it, agaeta@unisa.it, mgaeta@unisa.it
†Dip. di Studi e Ricerche Aziendali, Management & Information Technology, University of Salerno, Fisciano, Italy
Email: loia@unisa.it
‡Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada
Email: reformat@ualberta.ca

Abstract—We present an application of Granular Computing and Three-way decisions to intelligence analysis. In particular we extend the Analysis of Competing Hypotheses with an additional perspective devoted to support analysts in reasoning with groups of hypotheses that can be equivalent on the basis of partial and incomplete evidence, and in classifying these groups of hypotheses with respect to a decisional attribute of interest for the analyst, such as dangerous or safe. Creating and reasoning with granules and multi-level granular structures give to our approach an added value when dealing with a large number of evidence and hypotheses. Three-way decision making offers the possibility of a rapid understanding of how granules of hypotheses approximate a class of dangerous hypotheses, with clear benefits when analysts have to take decision on classifying a group of hypotheses or setting a proper level of attention to group of equivalent hypotheses.

I. INTRODUCTION

Intelligence analysis refers to a complex process devoted to gather information form different sources, and apply individual and collective cognitive methods to weigh data and evaluate hypotheses. The Analysis of Competing Hypotheses (ACH) of Heurer [1] is an important tool for the development of an intelligence analysis methodology. ACH supports analysts in the evaluation of a set of competing hypotheses against a set of evidence (that include logical arguments or assumptions) that can support or contradict each hypothesis. The objective is to proceed by rejecting or eliminating hypotheses, while tentatively accepting only those hypotheses that cannot be refused. As claimed by Heurer, ACH will not always yield the right answer but it can help analysts to overcome some cognitive limitations and biases. ACH forces analysts to clearly declare their assumptions and makes very easy tracing back from a decision to arguments, evidence and assumptions behind the decision. Current threats such as terrorism, cyber-terrorism and natural hazards, however, pose new challenges to intelligence analysis. As mentioned in the 9/11 commission report [2] it appears crucial to find a way of routinizing, even bureaucratizing, the exercise of imagination also by generating and analyzing hypotheses that can be considered very unlikely or can have a wide set of variations. This produces as consequence that the number of relations between evidence and hypotheses can quickly increase when a new hypothesis (with its variations) is formulated or a new evidence is gathered. The result is that human analysts have difficulties to apply ACH and reason on every single hypothesis. Moreover, in most real cases availability and quality of information is not so detailed and precise to allow reasoning on a specific single hypothesis and human analysts need to take into considerations groups of hypotheses that can be equivalents with respect to the available evidence. Approximate reasoning and Granular Computing (GrC) offer a possible solution for this issue. GrC is an information-processing paradigm focused on representing and processing basic chunks of information, namely information granules, and finds its origin in the works of Zadeh [3] that defines a granule as a clump of points (objects) drawn together by indistinguishability, similarity, proximity or functionality. Granules can be decomposed into smaller or finer granules called subgranules. Granules and subgranules can be organized by means of levels, hierarchies and granular structures [4]. The objective of our study is to enhance ACH tools with the application of GrC (in the formulation proposed by Yao [5] and based on Rough Sets [6]), and three-way decisions [7] in order to support analysts in:

• analysis and reasoning with granules, where a granule is a group of hypotheses that are indistinguishable with respect to available evidence, and
• taking decisions on the classification of such granules with respect to a decisional attribute of interest for the analysts, e.g. if the group of hypotheses can be classified as dangerous / safe or requiring / not requiring actions.

The availability of different levels of granulations can support analysts in reasoning with a very few set of evidence (e.g. 1 out of n) and understand what happens when other evidences could be available. The paper is organized as follows. Section II gives basic notions on ACH and three-way decisions with probabilistic rough sets. Section III presents our approach that basically consists of an enhancement of the analysis phase of ACH with a process of granulation, the creation of a multi-level granular structure in the form of a lattice of partitions, and the application of the three way decision making. Section

978-1-5090-1897-0/16/$31.00 ©2016 IEEE
IV presents a step-by-step example and section V reports a discussion and future works.

II. BACKGROUND

A. Analysis of Competing Hypotheses

The ACH process is detailed in [1] and consists of the following steps:

1) list all hypotheses and evidences (including assumptions and logical deductions).

2) Diagnostics Using a matrix, the analyst applies evidence e against each hypothesis h in an attempt to disprove as many theories as possible. An example of ACH matrix is as follows:

\[
\begin{array}{cccc}
\text{ACH} & h_1 & h_2 & \ldots & h_i \\
e_1 & + & NA & \ldots & - \\
e_2 & ++ & - & \ldots & - \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
e_j & -- & + & \ldots & +
\end{array}
\]

(1)

3) Refine the table - add, remove or merge hypotheses.

4) Analyse - analyst then seeks to draw tentative conclusions about the relative likelihood of each hypothesis, about the hypotheses most strongly supported, and so.

5) Sensitivity analysis - The analyst tests the conclusions using sensitivity analysis, which weighs how the conclusion would be affected if key evidence or arguments were wrong.

6) Report conclusions.

As mentioned, the usual way of reasoning in ACH is to refuse hypotheses and draw tentative conclusions about the relative likelihood. ACH does not consider the impact of hypotheses and this may lead to a risky situation where an analyst pays less attention to a set of refused hypotheses independently of their potential effects. As we will see in section III our approach includes this additional perspective of analysis supporting reasoning with group of hypotheses and considering a tentative classification of these group of hypotheses with respect to a decisional attribute chosen by the analyst.

B. Three-way decisions and probabilistic Rough Sets

A three-way decision model is an extension of the commonly used two-way, binary-decision model with an added third option and may be built with a fundamental notion of a trisection or a tri-partition of a universal set. Before introducing basic notions on three-ways decisions, we introduce first the concept of approximation in traditional rough set theory.

Let \( I = (U, A) \) be an information system, where \( U \) is a non-empty set of finite objects (the universe) and \( A \) is a non-empty finite set of attributes such that \( a : U \rightarrow V_a \) for every \( a \in A \), where \( V_a \) is the set of values that \( a \) can take. An information table assigns a value \( a(x) \) from \( V_a \) to each attribute \( a \) and object \( x \) in the universe \( U \). Given \( E \subseteq A \), we can define an equivalence relations \( IND(E) = \{(x, y) \in U \times U|\forall a \in E, a(x) = a(y)\} \). \( IND(E) \) states that \( x \) and \( y \) are indiscernible (or indistinguishable) by attributes from \( E \).

In other words, an equivalence relation can be defined based on a set of attributes in an information table so that two objects are equivalent if and only if they have the same value on every attribute. Given an equivalence relation \( E \), the equivalence classes of the E-indiscernibility relation are denoted as:

\[
[x]_E = \{y|y \in U, xEy\}
\]

(2)

Suppose \( H \subseteq U \) is a set of objects we want to describe, or approximate, with the equivalence classes. With rough sets we can approximate \( H \) by constructing its lower and upper approximations:

\[
apr(H) = \{x|x \in U, [x]_E \subseteq H\}
\]

(3)

\[
apr(H) = \{x|x \in U, [x]_E \cap H \neq \emptyset\}
\]

(4)

(3) and (4) can be also interpreted in terms of regions:

\[
POS(H) = apr(H)
\]

(5)

\[
NEG(H) = U - apr(H) = \{x|x \in U, [x]_E \cap H = \emptyset\}
\]

(6)

\[
BND(H) = apr(H) - apr(H) = \{x|x \in U, [x]_E \cap H \neq \emptyset [x]_E \notin H\}
\]

(7)

(5) is the positive region and includes all the equivalence classes that can be positively classified as belonging \( H \), (6) is the negative region and includes objects that can be definitely ruled out as members of \( H \) and (7) is the boundary region consisting of objects that can neither be ruled in nor ruled out as members of the target set \( H \).

In [7] it is provided a detailed description of the three-way decisions. In summary with respect to the three regions of \( H \), one can introduce three-way decision rules, namely, positive rules for accepting an object to be a member of \( H \), negative rules for rejecting, and boundary rules for deferring a definite decision. If \( [x]_E \subseteq POS(H) \) we decide to accept \( x \) as member of \( H \), if \( [x]_E \subseteq NEG(H) \) we decide to reject \( x \) as member of \( H \), and if \( [x]_E \subseteq BND(H) \) we decide to defer the decision. Now let us review (5), (6), and (7) in the perspective of probabilistic relationship between equivalence classes and target set \( H \) as defined in [8]. Let \( P(H|[x]_E) \) be the conditional probability of an object belonging to \( H \) given that the object belongs to \( [x]_E \). This probability can be estimated as \( P(H|[x]_E) = \frac{|H \cap [x]_E|}{|x|} \) where \(|.|\) is the cardinality. In terms of this conditional probability, the three areas can be formulated as follows:

\[
POS(H) = \{x|x \in U, P(H|[x]_E) = 1\}
\]

(8)

\[
NEG(H) = \{x|x \in U, P(H|[x]_E) = 0\}
\]

(9)
With the traditional rough set model, the decisions of acceptance and rejection are made without any error, we defer a decision if the conditional probability is between 0 and 1. In the three-way decision based on probabilistic rough sets we can introduce some tolerance of error choosing a pair of thresholds \( \alpha \) and \( \beta \) with \( \alpha > \beta \) and introduce three probabilistic regions:

\[
\begin{align*}
POS(H) &= \{x \mid x \in U, \ P(H[x]|E) \geq \alpha\} \\
NEG(H) &= \{x \mid x \in U, \ P(H[x]|E) \leq \beta\} \\
BND(H) &= \{x \mid x \in U, \ \beta < P(H[x]|E) < \alpha\}
\end{align*}
\]

With the traditional rough set model, the decisions of acceptance and rejection are made without any error, we defer a decision if the conditional probability is between 0 and 1. In the three-way decision based on probabilistic rough sets we can introduce some tolerance of error choosing a pair of thresholds \( \alpha \) and \( \beta \) with \( \alpha > \beta \) and introduce three probabilistic regions:

\[
\begin{align*}
POS(H) &= \{x \mid x \in U, \ P(H[x]|E) \geq \alpha\} \\
NEG(H) &= \{x \mid x \in U, \ P(H[x]|E) \leq \beta\} \\
BND(H) &= \{x \mid x \in U, \ \beta < P(H[x]|E) < \alpha\}
\end{align*}
\]

Fig. 1 shows the regions we can define on the basis of (a) rough sets and (b) probabilistic rough set models. The values of \( \alpha \) and \( \beta \) can be evaluated with the formulas discussed in [7] based on the Bayesian decision theory for classification.

### III. 3W-ACH

The approach for enhancing ACH with three-way decisions, shortly called 3W-ACH, is shown in Fig. 2. We aim at supporting analysts in reasoning with group of hypotheses in the presence of an incomplete set of evidence, and taking rapid and informed decisions about acceptance or rejection of group of hypotheses with respect to a decisional attribute that is of interest for the analysts. Analysts have knowledge and experience to classify each hypothesis with respect to a decisional attribute, e.g., dangerous or safe. In real scenarios, however, just few evidence can be available and a high number of hypotheses can be equivalent, making classification of the hypotheses not so straightforward and not always possible (deferring decision). GrC and three-ways decision are employed in our approach that foresees the following phases:

- as for the traditional ACH, an analyst has to formulate hypotheses and collect evidences in order to prepare a ACH matrix such as (1). For our objective, we add a row and a column to matrix (1) providing respectively a binary decisional attribute \( d \) for each hypothesis (Dangerous or Safe) and a binary decisional attribute \( c \) (Credible, NotCredible). This results in a data structure such as (14) where \( d = \{D, S\} \) and \( c = \{C, NC\} \)

\[
\begin{pmatrix}
\begin{array}{ccc}
h_1 & h_2 & \ldots & h_i & c \\
+ & + & \ldots & - & C \\
++ & + & \ldots & - & NC \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
- & - & \ldots & + & C \\
D & S & \ldots & S
\end{array}
\end{pmatrix}
\]

- Granulation and multi-level granular structures. The first refers to the process of creating granules starting from a data structure such as (14), the second refers to the construction of a multi-level structure by putting together simple granules. From a data structure such as (14) we can create two kinds of granules: granules of evidence \( e \) or granules of hypotheses \( h \). In the following section IV we are going to use rough set as a formalism for GrC so we will consider indistinguishability as a criterion for granulation, so that \( h \) groups hypotheses that are equivalent with respect to a set of attributes-values. However, an analyst can also define and use another measure of similarity as criterion of granulation, or use alternative approaches to group hypotheses on the basis of feature selection, such as the GA-based attribute clustering process proposed in [9]. In all the cases, a granule \( h \) can be refined or abstracted. A refinement relation produce finer granules \( h_i \preceq h \) while a coarser relation produce more abstract granules \( h_j \succeq h \). Hierarchies of granules such as the one shown in Fig. 3 can be constructed via refinement or abstraction relations.

- Abductive reasoning, that is a form of reasoning that starting from some observations/evidence allows to infer hypotheses that can explain those observations. A form of abductive reasoning can be supported by granules and granular structures. Creating granules of hypothesis that are equivalent or similar on the basis of a subset of evidence is yet a simple form of abductive reasoning since allows an analyst to infer groups of hypotheses that are equivalent or similar with respect to the available (usually
consistent, 1 = consistent. The hypotheses and evidence with binary values, i.e. 0 = in-
A. Granules and Granular structures
However, the benefits of our approach in cases with high
presents the classification of granules. The case in the illus-
the creation of granules and granular structures. The second
ACHt
I. Let us consider also the transposed matrix
II.
We divide the example in two phases. The first relates to
creation of granules and granular structures. The second
details our approach.
IV. ILLUSTRATIVE EXAMPLE
We divide the example in two phases. The first relates to
the creation of granules and granular structures. The second
presents the classification of granules. The case in the illus-
example reports a reduced number of evidence and
hypotheses to allow a better comprehension of all the steps.
However, the benefits of our approach in cases with high
numbers of alternative hypotheses and evidences can be clearly
understood.
A. Granules and Granular structures
Let us consider for illustrative purposes a case with 8
hypotheses and 3 evidence with binary values, i.e. 0 = in-
consistent, 1 = consistent. The ACH matrix is shown in table
I. Let us consider also the transposed matrix ACHt in table
II.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>ACH matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>0 0 0 1 1 1 1 1</td>
</tr>
<tr>
<td>e2</td>
<td>0 0 1 1 0 1 1 1</td>
</tr>
<tr>
<td>e3</td>
<td>0 1 0 1 0 1 0 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>ACHt matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>e2</td>
</tr>
<tr>
<td>h1</td>
<td>0 0 0</td>
</tr>
<tr>
<td>h2</td>
<td>0 0 1</td>
</tr>
<tr>
<td>h3</td>
<td>0 1 1</td>
</tr>
<tr>
<td>h4</td>
<td>0 1 1</td>
</tr>
<tr>
<td>h5</td>
<td>1 0 0</td>
</tr>
<tr>
<td>h6</td>
<td>1 0 1</td>
</tr>
<tr>
<td>h7</td>
<td>1 1 0</td>
</tr>
<tr>
<td>h8</td>
<td>1 1 1</td>
</tr>
</tbody>
</table>

From a rough set perspective, ACH and ACHt can be
considered as two information tables. For ACH the universe
of objects U consists of evidence and the attributes are
hypotheses. For ACHt it is the contrary. We can apply the
notions of granulation by partition defined by Yao [5]. We
can define two elementary granules on the basis of ACH
and ACHt that are formalized respectively by the following
equivalence relations:

\[
[e]_H = \{e' | e' \in U, f(e) = f(e')\}
\]

\[
[h]_E = \{h' | h' \in U, f(h) = f(h')\}
\]

where f is in both the cases an information function for
each attribute.
(15) and (16) are granules of indistinguishable objects,
and the meaning of the two granules is as follows: (15)
means that given a set H of hypotheses the evidence e and
e' are equivalent because have the same value on every
hypothesis h \in H, (16) means that given a set E of evidence
the hypotheses h and h' are equivalent because have the
same value on every evidence e \in E. Let concentrate our
attention on granules of equivalent hypotheses (16) and on
the information table ACHt of Table II.
Consider the following subsets of attributes for ACHt:
A₁ = \{e₁\}, A₂ = \{e₁, e₂\}, and A₃ = \{e₁, e₂, e₃\} that satisfy
a nested structure, A₃ ⊆ A₂ ⊆ A₁. We define the equivalence
relations on this sequence of subsets I = E₄/E₃ ⊆ E₃/E₂ ⊆ E₂/E₁ ⊆ E₁ = U × U, and derive the following multi-level
granular structure:

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>MULTI LEVEL GRANULAR STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>L4</td>
<td>{h₁, h₂, h₃, h₄, h₅, h₆, h₇, h₈}</td>
</tr>
<tr>
<td>L3</td>
<td>{h₁, h₂, h₃, h₄}, {h₅, h₆, h₇, h₈}</td>
</tr>
<tr>
<td>L2</td>
<td>{h₁, h₂}, {h₃, h₄}, {h₅, h₆}, {h₇, h₈}</td>
</tr>
<tr>
<td>L1</td>
<td>{h₁}, {h₂}, {h₃}, {h₄}, {h₅}, {h₆}, {h₇}, {h₈}</td>
</tr>
</tbody>
</table>

where granules at the top (L4) correspond to the equiva-
ence relation E₀, L₃ corresponds to the equivalence relation
E₄/E₃ that are the subsets of hypotheses that are equivalent
considering only the evidence e₁, and so on. It is clear that
L₁ considers all the attributes of our information table ACHt
and provides the finest granularity (allowing to discern all the
hypotheses) while L₄ provides the coarsest one.
If we consider all the possible subsets of attributes of the
information table ACHt the resulting multi level granular
structure is a lattice of partitions (granules) such as the one of
Fig. 4, where to simplify the comprehension we do not report
L₄ and L₁, and for each partition we report the values of the attributes.

By inspecting this structure an analyst has a clear view
of the hypotheses that can be equivalent on the basis of the
partial set of evidences. If instead of indistinguishability we
used other similarity metrics, Fig. 4 gives a view of similar
hypotheses. Moreover, it is also easy to reason on refusing
hypotheses (that is the traditional way of reasoning of the ACH
process) on the basis of the available evidence. For instance,
it easy to determine that on the basis of the incomplete set of evidence \( \{e_1, e_3\} \) the granule \( \{h_1, h_3\} \) has to be refused since both the evidence are inconsistent. Besides refusing hypotheses, we want to give also an additional information: given information on classification of each hypothesis with respect to its effect and classification of each evidence with respect to its credibility, we want to classify the granules.

### B. Classifying granules

The classification of granules is not trivial and, at the same time, it is very important for an analyst in order to set a proper level of attention also for refused hypotheses or unlikely ones. Three-way decisions can support this.

Let us add a decisional attribute to \( ACH \) and \( ACH_t \) to classify evidence and hypothesis with respect to their credibility or effects. Tables I and II becomes two decision tables (that is an information table with the addition of a decisional attribute) such as: These two tables can be derived from (14)

#### TABLE IV

<table>
<thead>
<tr>
<th>h1</th>
<th>h2</th>
<th>h3</th>
<th>h4</th>
<th>h5</th>
<th>h6</th>
<th>h7</th>
<th>h8</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>e2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>C</td>
</tr>
<tr>
<td>e3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>NC</td>
</tr>
</tbody>
</table>

#### TABLE V

<table>
<thead>
<tr>
<th>e1</th>
<th>e2</th>
<th>e3</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>h1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>h2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>h3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>h4</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>h5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>h6</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>h7</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>h8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and \( c \) and \( d \) have the same meaning. Let us consider Table V and let \( D = \{h_1, h_3, h_4, h_6, h_7\} \) be the set of all dangerous hypotheses. Given the granules belonging to a partition we can evaluate if they belong to \( \text{POS}(D) \), \( \text{NEG}(D) \) or \( \text{BND}(D) \), that are the regions that approximate positively the set \( D \), negatively or advise for a deferring decision. Let us consider, for instance, the equivalence relation \( E_{\lambda_2} \) corresponding to the granular level \( L_2 \) of table III. Let fix \( \alpha = 0.63 \) and \( \beta = 0.25 \), that are the values used in the example of [7], that are in favour of deferment decisions and based on the assumption that the risk of assigning an object into \( \text{BND} \) is between a correct classification and an incorrect one. The universe of 8 hypotheses is partitioned in 4 granules, each of them with 2 hypotheses. Evaluating \( P(D|\{h\}_E) \) we have the following results:

#### TABLE VI

| \( D \cap \{h\}_E \) | \( P(D|\{h\}_E) \) |
|---------------------|---------------------|
| \( \{h_1, h_2\} \)  | 1                   |
| \( \{h_3, h_4\} \)  | 2                   |
| \( \{h_5, h_6\} \)  | 1                   |
| \( \{h_7, h_8\} \)  | 1                   |

Thus \( \text{POS}(D) = \{h_3, h_4\} \), \( \text{BND}(D) = \{h_1, h_2\} \cup \{h_5, h_6\} \cup \{h_7, h_8\} \) meaning that at \( L_2 \) the granule \( \{h_3, h_4\} \) is an approximation of the dangerous hypotheses. If we evaluate \( \text{POS}(D) \), \( \text{NEG}(D) \) and \( \text{BND}(D) \) for all the levels of a multi level granular structure, we can enhance the lattice of partitions of Fig. 4 with information on how each granule approximates the class of dangerous hypotheses. This is shown in Fig. 5. It is easy to understand that what we have described so far for granules of hypotheses, such as (16), on the basis of the decision table VI, can be done also for granules of equivalent evidence, such as (15), on the basis of the decision table V. In this case we can create lattice of partitions of granules of evidence and understand how these granules approximate a class of credible evidence.

The availability of information on credibility of evidence can support the analyst in reasoning in incremental way. The analyst may start with the availability of a single evidence and next refining the granules by including additional evidence that can lower or improve the overall credibility. An example is shown in Fig. 6 where an analyst starts to reason on the basis of one evidence with granules at \( L_3 \) of our example. He can refine the granule by adding credible evidence (e.g. \( L_2 \cdot e_1 \cdot e_2 \)) or lowering the credibility of the evidence (e.g. \( L_2 \cdot e_1 \cdot e_3 \)). In our example \( h_1 \), \( h_3 \) and \( h_4 \) demand particular attention.

This kind of relation can be also mapped on the lattice of partition. In the case of Fig. 5 the two relations of Fig. 6 are shown with colours red and blue.
to extend our approach with the addition of methods to derive evidence from the collective observations of communities of professionals and citizens on site using, for instance, social approaches that takes into account contextual information [16].

With respect to this last point, however, we need to consider that evidence gathered from collective observations are usually less precise and more vague because of a lack of knowledge of citizens on the phenomena and this can be dangerous when performing intelligent analysis. To manage this issue, we aim at integrating in our approach the concept of fuzzy signature, originally proposed in [17] and that can be used to argument the evidence in an ACH process on the basis of a collective perception of a phenomena using belief theory in a way similar to the one proposed in [18].

V. DISCUSSION AND FUTURE WORKS

The application of GrC and three-way decisions we presented in this paper is devoted to add a different and complementary perspective of analysis to traditional ACH process, supporting reasoning with groups of hypothesis to understand if can be classified according to a classification attribute of interest for the analyst. This is not in contrast with the formal objective of ACH, that is to refuse hypotheses, and with the estimation of relative likelihood of hypotheses. To the best of our knowledge, this is one of the first application of GrC to ACH. Other works enhance ACH with the adoption of belief theory and subjective logic [10] where formal calculus, such as the subjective logic, is used to make recommendations on likelihoods of hypotheses based on uncertain knowledge about the evidence. With similar objectives, [11] presents and discusses a Bayesian extension of ACH in terms of multinomial Dirichlet hierarchical model, and the adoption of Bayesian networks to abstract and generalize ACH table is proposed in [12]. In terms of tools to support ACH, the kinds of analysis we envisage are not developed. Besides the software developed by Palo Alto Research Center (PARC) in collaboration with Richards J. Heuer (http://www2.parc.com/istl/projects/ach/ach.html), that support usual ACH process, one of the most comprehensive software solution is developed by Globalytica (http://www.globalytica.com/thinksuite-html/) and includes also tools for hypotheses generation. This last feature is interesting and is one of our future works that go in three directions:

- to evaluate our approach into a concrete scenario. Crime investigations, evidence based medicine and cyberwarfare scenarios [13] appear to be suitable candidates. Also smart commerce scenarios [14] based on recognition and assessment of situations of interest for the consumers (hypotheses) can be useful,

- to include hypotheses generation in our process. Morphological analysis is a method that can be used to generate plausible hypotheses and has been also applied in combination with ACH [15]. Usually, it generates an high number of plausible hypotheses. This aspect fits well with our application of GrC since we can easy manage this issue, with the application of the equivalence relations and other rough sets concept such as reduc,